Ages of Origin and Destination
for a Difference in Life Expectancy

Salience of the Life Expectancy Measure

The life table provides a convenient, comprehensive and self-contained summary of mortality conditions prevailing in an actual or hypothetical population. The relations among its columns and parameters have formed one of the most fruitful traditions of mathematical population research.

Of all the summary measures that can be derived from a life table, the expectation of life (or life expectancy) is perhaps the most well-known, widely-used, widely-cited and widely-studied statistic. For any age x (most frequently, age zero or birth) e_x reports the mean number of person-years each person attaining age x can expect to live, given the mortality rates observed throughout the entire life table. Two different life tables, reflecting conditions in two different populations or in a single population at two different times, ordinarily report two different expectations of life at any age.

Briefly, this expectation of life at any age depends on two life table measures, survivors to exact age l_x and total person-years lived ${}_nL_i$ in age groups above age x. Both measures derive from observed risks of death in each age group (Chiang 1984, Preston et al 2001). The expectation of life at any age x simply divides the person-years left to live above age x by the number of survivors to that age who are left to live them:

$$e_x = \frac{\sum_{i=x,n}^{\Omega} {}_nL_i}{l_x}. \qquad (1)$$

The complex relation between actual mortality changes at various ages and a resulting change in life expectancy (Pollard 1982, Vaupel 1986) has given rise to much analysis of the underpinnings of the e_x statistic (Pollard 1988, Vaupel & Canudas-Romo 2002, 2003). This study presents a straightforward and potentially useful way to think about the components of a difference in life expectancies in two complementary senses. All formulas present the discrete case most directly applicable to observed empirical data. Conversion to the more abstract continuous case would be a simple matter for those who prefer more precise mathematical expressions. On the one hand, we explore the ages at which a difference in life expectancy originates. On the other hand, we consider the ages at which a difference in life expectancy is actually lived. Centered on the concept of temporary life expectancies within age groups, we show that these two perspectives on the origin and destination of a difference in life expectancy represent orthogonal dimensions of the same underlying decomposition approach.

To simplify our descriptions, we introduce some standard extensions of basic life table notation. A bar over l_x or e_x indicates the arithmetic mean of l_x or e_x values from different life tables, as defined by equation 2:

$$\bar{l}_x = \left(\frac{l_x^a + l_x^b}{2}\right); \bar{e}_x = \left(\frac{e_x^a + e_x^b}{2}\right). \qquad (2)$$

As defined by Arriaga (1984), the ${}_ne_x$ measure of temporary life expectancy represents person-years lived within a specific age interval, per person alive at the start of the interval (dividing ${}_nL_x$ by l_x as in equation 3 below). It also can represent the ratio of the probability of dying in the interval to the average force of mortality during the interval (dividing ${}_nq_x$ by ${}_nm_x$ as in equation 3).

$$ {}_ne_x = \frac{{}_nL_x}{l_x} = \frac{{}_nq_x}{{}_nm_x}. \qquad (3)$$

We also define an extension of temporary life expectancy, called conditional temporary life expectancy, as person-years lived in some age interval above the age group x to $x+n$, per person alive at exact age x, the "baseline age" for this partial measure. Conditional temporary life expectancy can be written:

$$ {}_ne_{i|x} = \frac{{}_nL_i}{l_x}, i \geq x. \qquad (4)$$

Since all conditional temporary life expectancies starting at the same baseline age share the same denominator l_x, the sum of all these terms (including the unconditional or "ordinary" temporary life expectancy, when i equals x) is simply the expectation of life at age x as shown in equation 5 below.

$$ e_x = \frac{\sum_{i=x,n}^{\Omega} {}_nL_i}{l_x} = \sum_{i=x,n}^{\Omega} {}_ne_{i|x}. \qquad (5)$$

Ages Where a Difference in Life Expectancy Originates

Several popular decompositions of a difference in life expectancy ask about the origin of the difference. That is, they allocate a difference across ages where mortality differences first influence the number of person-years to be lived at subsequent ages (Shkolnikov, Valkonen, Begun & Andreev, 2001). This *origin-decomposition* approach includes the well-known methods of Arriaga (1984), Andreev (1982) and Pressat (1982). An origin-decomposition breaks down a difference in life expectancy at birth according to the ages at which the lives are originally saved (but see Vaupel & Yashin 1987 on the thorny issue of “repeated lifesaving”). For simplicity we consider the origin-decomposition proposed by Andreev (1982), who calculated two versions of an age-specific measure ${}_n\varepsilon_x$ and then averaged the two:

$$ {}_n\varepsilon_x^a = \left[l_x^a \cdot (e_x^a - e_x^b)\right] - \left[l_{x+n}^a \cdot (e_{x+n}^a - e_{x+n}^b)\right], \qquad (6a)$$

$$ {}_n\varepsilon_x^b = \left[l_x^b \cdot (e_x^b - e_x^a)\right] - \left[l_{x+n}^b \cdot (e_{x+n}^b - e_{x+n}^a)\right], \qquad (6b)$$

$$ e_0^b - e_0^a = \sum_{x=0,n}^{\Omega} \left({}_n\varepsilon_x\right) = \sum_{x=0,n}^{\Omega} \left(\frac{{}_n\varepsilon_x^b - {}_n\varepsilon_x^a}{2}\right). \qquad (6c)$$

The average in the final term of equation 6c subtracts rather than adds because the reversed order of subtraction of life expectancies in 6a and 6b give the two partial terms opposite signs. Taking the average of l_x values (see equation 2 above) from the outset seems simpler, as shown in equation 7 below, which yields identical results:

$$e_0^b - e_0^a = \sum_{x=0,n}^{\Omega} \left({}_n\varepsilon_x \right) = \sum_{x=0,n}^{\Omega} \left(\left[\bar{l}_x \cdot \left(e_x^b - e_x^a \right) \right] - \left[\bar{l}_{x+n} \cdot \left(e_{x+n}^b - e_{x+n}^a \right) \right] \right) \qquad (7)$$

For each age group, one finds the weighted difference in person-years lived above age x (the first term) and then subtracts the weighted difference lived above age $x+n$ (the second term) to attribute ${}_n\varepsilon_x$ to the age group itself as a remainder. The weights are simply averages of the l_x values in the populations. For the last open-ended interval in a life table, the second term in equation 7 equals zero because e_{x+n} does not exist.

To illustrate this origin-decomposition of differences in life expectancy at birth, consider life table values for the black and white male and female populations of the United States in the year 2000 as shown in Table 1.

Table 1 Here

Table 1 shows proportions of each race/sex group surviving to specified ages, and also the person-years lived in each age interval including the final open-ended interval. As suggested by equation 1 above, the sum at the bottom of each ${}_nL_x$ column gives the expectation of life at birth. Black males had the lowest life expectancy at birth in 2000. Differences in e_0 by race and sex were roughly equal in size and additive. That is, both black females and white males could expect about six more years of life based on 2000 mortality rates than could black males. White females had the highest expectation of life.

Andreev's ${}_n\varepsilon_x$ age decomposition allocates differences in life expectancy according to the age group where the mortality difference first occurs, so we classify this approach as an origin-decomposition. It aims to identify the age groups where changes in life expectancy originate. Figure 1 concentrates attention on black men because their survival rates were worst in 2000, showing a sex difference on one hand (black men compared to black women) and a race difference on the other (black men compared to white men).

Figure 1 Here

Compared to white men, black men in the United States in 2000 experienced much higher rates of infant death, accounting for over one-tenth of the total difference in life expectancy at birth. On the other hand, higher death rates for black than for white boys between ages 1 and 15 had very little impact on the difference in expectation of life, in part because the rates themselves were so low at these ages for both groups, and in part because the difference in death rates by race was relatively small. Age-specific contributions to the race difference in life expectancy for American men in 2000 increased gradually after childhood, however, and peaked in the late working ages (roughly 45 to 65) where almost half of the total difference in life expectancy at birth originated. Older ages contribute much less to the race difference in life expectancy, in part because there are fewer survivors left at these older ages to contribute person-years lived, and in part because reported mortality rates converge in old age for black and white men. This convergence has been the subject of intense interest, and its significance continues to be debated.

Compared to black women, black men in the United States in 2000 exhibited a life expectancy disadvantage that very closely resembled the race contrast between black and white men in both absolute magnitude (6.75 years for the sex contrast versus 6.60 years for the race contrast) and in its ages of origin. The major differences between the two age patterns in Figure 1 are that infancy contributed much less to the sex contrast while young adulthood (when males of all backgrounds seem to exhibit anomalously high mortality rates) contributed more to the sex contrast. Mortality differences originating in late middle age and early retirement dominate both age and sex contrasts. At these ages most people are still alive but a lifetime of various social, economic and health effects begin to take their cumulative toll and death rates rise prematurely for disadvantaged groups. The sex difference in life expectancy also owes more to mortality differences in old age than does the race difference.

Destination Ages Where a Difference in Life Expectancy is Lived

For conventional period life table analysis, origin-decomposition often proves most useful because such life tables represent fictional extrapolations from actual mortality conditions to a synthetic "life table population" that only exists conceptually. Naturally, we most commonly wish to understand how these actual conditions affect the resulting thought-experiment that is the life table.

However, for many reasons we also may be interested in the ages at which a difference in life expectancy is lived or in the age structure of the life table population, also represented by ${}_nL_x$ values. For example, how mortality trends or differences affect the age structure of a population's labor force may interest economists. How a difference in mortality by sex affects the sex ratio in different age groups may interest family scholars. In the case of cohort life tables that follow actual generations over time, the destinations at which person-years are lived also correspond to real-world circumstances, so that destination-decomposition may be of intrinsic interest. Similarly, simulation exercises such as experiments with stable population theory (where all features of the life table may be hypothetical) also may find the destination ages where a difference in life expectancy is lived to be of equal importance with the origin ages where mortality differences give rise to such effects.

In its simplest form, destination-decomposition operates directly on ${}_nL_x$ values, taking advantage of the fact that by re-arranging equation 3 above, each ${}_nL_x$ can be expressed as the product of survivors to the start of the age group l_x and the temporary life expectancy ${}_ne_x$. Use of a new age subscript i emphasizes the distinction between ages of origin and ages of destination as alternate decompositions. With two terms, we may consider ordinary component decomposition (Das Gupta 1978, 1994). Equation 8 below includes two components.

$$e_0^b - e_0^a = \sum_{i=0,n}^{\Omega}\left({}_nL_i^b - {}_nL_i^a\right) = \sum_{i=0,n}^{\Omega}\left(\bar{l}_i \cdot ({}_ne_i^b - {}_ne_i^a)\right) + \left({}_n\bar{e}_i \cdot (l_i^b - l_i^a)\right) \qquad (8)$$

The first or direct component (an average of l_i values times the difference in ${}_ne_i$) measures how differences in mortality rates within the age range affect person-years lived in the age group. Averaging l_i values avoids taking one population or the other as a standard or baseline. The second or indirect component (an average of ${}_ne_i$ values times the difference in l_i) measures the contribution from cumulative mortality differences at younger ages to a difference in person-years lived in the interval, because such earlier differences affect the share of the population reaching the age group and so change the person-years lived within it.

Figure 2 Here

Using the same data from Table 1 to compute the destination-decomposition of differences in ${}_nL_i$ values directly, Figure 2 displays a completely different way of thinking about a decomposition of a difference in life expectancy in contrast to Figure 1. The two alternative decompositions represent answers to different questions. An origin-decomposition tells us at which ages a difference in life expectancy originates. A destination-decomposition tells us at which ages a difference in life expectancy is lived. Both decompositions add up to the total difference in life expectancy, but Figure 2 attributes much less of the difference to young ages where differences in person-years lived within the age groups were quite small, and places greater emphasis on older ages at which differences in death rates from earlier ages result in new person-years of life.

Although the two decompositions produce radically different answers to different questions, obviously they also are related to one another. Both are built up from exactly the same building blocks, namely the values of survivors to each age l_x and the person-years lived in each age group ${}_nL_x$. In proper dialectic form, the final section of this analysis provides the synthesis that connects the two.

<u>Bridging between Origin and Destination Decomposition</u>

We may expand an origin-decomposition such as Andreev's into a series of terms involving temporary and conditional temporary life expectancy as described above. When we do this, the first bracketed term from equation 7 becomes a difference in temporary life expectancy ${}_ne_x$ weighted by the averaged l_x value (a "direct" effect), plus a series of differences in conditional temporary life expectancies for older age groups also weighted by the averaged l_x value (the second bracketed term in equation 9a below). The direct effect (the first bracketed term in equation 9a) closely resembles the direct effect specified by Arriaga (1984) except that Arriaga's method did not average the l_x values, privileging one of them as a baseline. Note that this direct effect is identical to the direct effect specified in equation 8 above for destination-decomposition.

$$e_0^b - e_0^a = \sum_x \left({}_n\varepsilon_x\right) = \sum_{x=0}^{\Omega}\left[\left[\bar{l}_x \cdot \left({}_ne_x^b - {}_ne_x^a\right)\right] + \left[\bar{l}_x \cdot \sum_{i=x+n,n}^{\Omega}\left({}_ne_{i|x}^b - {}_ne_{i|x}^a\right)\right] - \left[\bar{l}_{x+n} \cdot \sum_{i=x+n,n}^{\Omega}\left({}_ne_{i|x+n}^b - {}_ne_{i|x+n}^a\right)\right]\right]. \quad (9a)$$

The second bracketed term from equation 7, when expanded in terms of conditional temporary life expectancy as shown in the third bracketed term of equation 9a above, contains no reference to the age group from x to $x+n$. However, it does contain a series of expressions for older age groups that match one-for-one with the equation's second bracketed term. The third term differs from the second term in that the baseline age becomes $x+n$ rather than x. Since the two summations cover the same ages, we may re-arrange the second and third terms as shown in equation 9b below:

$$e_0^b - e_0^a = \sum_x \left({}_n\varepsilon_x \right) = \sum_{x=0}^{\Omega} \left[\left(\bar{l}_x \cdot \left({}_n e_x^b - {}_n e_x^a \right) \right) + \sum_{i=x+n,n}^{\Omega} \left[\left(\bar{l}_x \cdot \left({}_n e_{i|x}^b - {}_n e_{i|x}^a \right) \right) - \left(\bar{l}_{x+n} \cdot \left({}_n e_{i|x+n}^b - {}_n e_{i|x+n}^a \right) \right) \right] \right]. \quad (9b)$$

Andreev's ${}_n\varepsilon_x$ expression for a difference in life expectancy originating in a given age group thus breaks down across all subsequent age groups using the terms in equation 9b above. We can see not only how much of the total difference in life expectancy originated in each age group, but also in which age groups that portion of the total difference subsequently was lived.

For example, consider the ${}_n\varepsilon_x$ age decomposition of the race difference in life expectancy between black men and white men in the United States in 2000, depicted in Figure 1 above. Appendix Table A distributes each difference originating in a specific age group across subsequent age groups where that difference would be lived out in the life table. The first cell with a value for each age group (row of Appendix Table A) shows the direct effect as the average of l_x values times the difference in ${}_ne_x$ values. Subsequent cells in that row contain the sequence of differenced, weighted ${}_ne_{i|x}$ differences as described in equation 9b. Each step of the summation over i yields a value for another age group in the row. Together, the values in each row sum to Andreev's ${}_n\varepsilon_x$ effect. Summing over x then produces the total difference in life expectancy.

In this example the total effect attributed to each age group involves some difference in person-years actually lived within that age group itself, but particularly for younger ages, most of the effect comes from the "echo" of this difference passing through subsequent age groups and being further amplified by differences in survival found there, analogous to Arriaga's (1984) description of direct and indirect effects. Of the 0.67 years of difference in life expectancy due to mortality differences between black and white male infants, for example, 0.08 years comes from the direct effect within infancy itself, and the remaining 0.59 years accumulate through the rest of the considered lifespan in the life table population. On the other hand, for ages 55 through 59, the age group where the largest single difference (0.77 years) originated, most of that difference is lived in the first few succeeding age groups because the proportions surviving to older age groups (l_x values) decline so rapidly at later ages. The values from Appendix Table A appear graphically in Figure 3 below.

Figure 3 Here

Similarly, Appendix Table B distributes each age-specific difference between black men and black women in the United States in 2000, showing the ages where the sex difference in life expectancy originating in each age group subsequently was lived. In Appendix Table B the sex difference in infant mortality produces much less of the total sex difference in life expectancy at birth (only a fifth of a year, compared to two-thirds of a year for the race difference in Table A). Again, most of this effect accumulates over the entire lifespan after infancy. On the other hand, the excess male mortality in young adulthood, already pointed out in Figure 1 above, produces extra years of expected life for black women compared to men that are lived mostly in the immediate succeeding ages--over half of this 0.40 difference in life expectancy is lived in the three decades between ages 20 and 50. The graphic representation of Appendix Table B appears in Figure 4 below.

Figure 4 Here

In addition to seeing the distribution of each origin-effect over subsequent age groups by looking "forward" along the rows for ages of origin in the Appendix tables or Figures 3 and 4, one also may look orthogonally "backward" up the columns for ages of destination instead. In each case, the sequence starts with weighted temporary life expectancy within the age group itself (the direct effects in the identical first terms of equations 7 and 8). The second term in equation 7 looks forward to older ages along rows in the appendix tables, while the second term in equation 8 looks backward to younger ages up columns in the tables.

The total for each column gives the total difference in person-years lived in each destination age group i to $i+n$. In effect, one sums on origin age x rather than on destination age i as in equation 9b above.

$$e_0^b - e_0^a = \sum_{i=0}^{\Omega}\left({}_nL_i^b - {}_nL_i^a\right) = \sum_{i=0}^{\Omega}\left[\left(\bar{l}_i \cdot \left({}_ne_i^b - {}_ne_i^a\right)\right) + \sum_{x=0,n}^{i-n}\left[\left(\bar{l}_x \cdot \left({}_ne_{i|x}^b - {}_ne_{i|x}^a\right)\right) - \left(\bar{l}_{x+n} \cdot \left({}_ne_{i|x+n}^b - {}_ne_{i|x+n}^a\right)\right)\right]\right]. \quad (10)$$

Summing over x as in equation 10, each iteration beginning from $x = 0$ contains a term for the current age group minus an equivalent term for the next age group. The next step in the summation then includes the earlier second term as the new first term. Each term beyond the first age group cancels out by being added once and subtracted once. The term added in the final iteration of the sum cancels out the "direct effect" or temporary life expectancy for the age group itself. Only the first term,

$$\bar{l}_0 \cdot \left({}_ne_{i|0}^b - {}_ne_{i|0}^a\right) = 1 \cdot \left(\frac{{}_nL_i^b}{1} - \frac{{}_nL_i^a}{1}\right),$$

"survives" the summation.

Thus equations 9b and 10 form a "bridge" between origin-decomposition and destination-decomposition of a difference in life expectancies. Tables A and B in the Appendix actually contain two-dimensional origin/destination decomposition matrices of differences in life expectancy, rather than single vectors that attribute the difference either to ages of origin or ages of destination. From these tables, one may inspect the share of any difference in life expectancy that originated in any particular age group, *and* that was lived in any other particular age group. Summing in one direction produces the origin-decomposition, while summing in the other direction produces the destination-decomposition. This two-dimensional tool for thinking about differences in life expectancy may offer new flexibility in analyzing changes and differences in the chances of survival in human populations.

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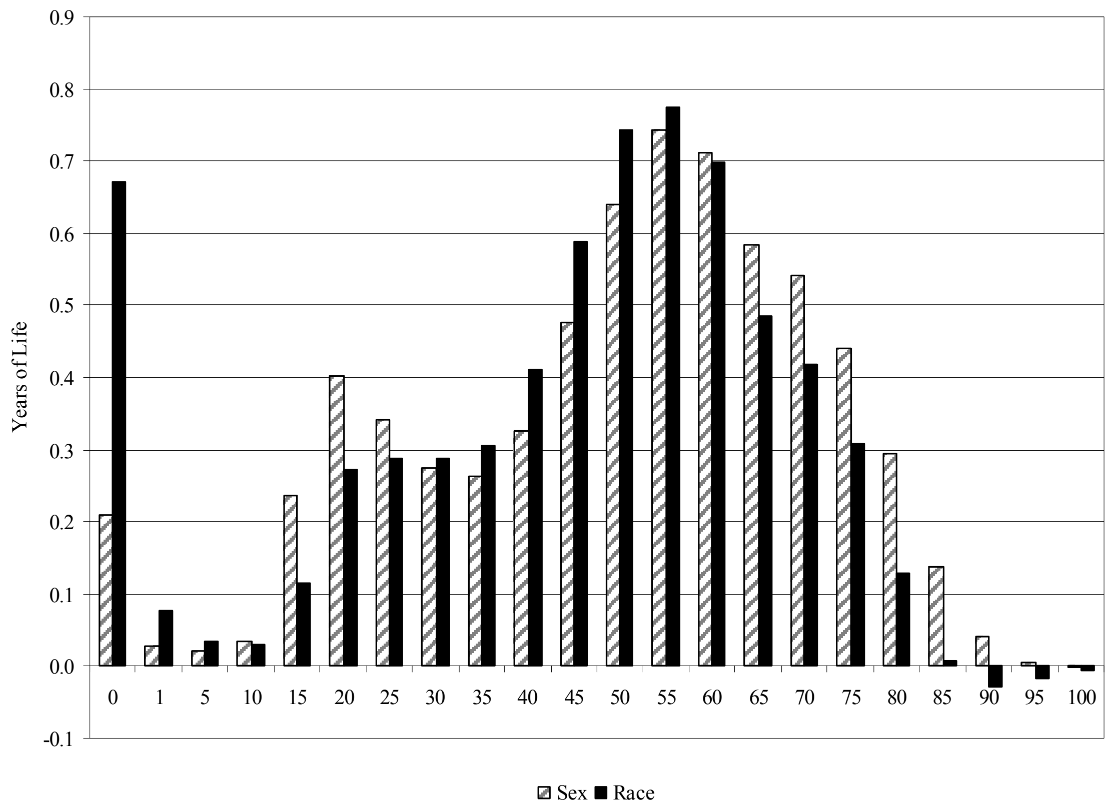
Table 1 - Survival Values by Race and Sex, United States 2000

Age x	Black Male l_x	Black Female l_x	White Male l_x	White Female l_x	Black Male $_nL_x$	Black Female $_nL_x$	White Male $_nL_x$	White Female $_nL_x$
0	1.000000	1.000000	1.000000	1.000000	0.986323	0.988863	0.994520	0.995500
1	0.984440	0.987330	0.993760	0.994870	3.931890	3.944600	3.971985	3.977035
5	0.982060	0.985350	0.992470	0.993860	4.906625	4.925205	4.960130	4.967590
10	0.980710	0.984300	0.991630	0.993200	4.900490	4.919035	4.956065	4.964405
15	0.979050	0.983210	0.990460	0.992430	4.881350	4.911105	4.942410	4.957535
20	0.972590	0.981000	0.986040	0.990460	4.835370	4.896300	4.914825	4.946955
25	0.961070	0.977300	0.979770	0.988310	4.774945	4.874740	4.883480	4.935595
30	0.948860	0.972320	0.973630	0.985860	4.711295	4.844540	4.851580	4.921870
35	0.935310	0.965060	0.966750	0.982680	4.636110	4.800870	4.811970	4.901955
40	0.918270	0.954620	0.957550	0.977770	4.533170	4.736175	4.756930	4.871595
45	0.893330	0.938790	0.944410	0.970440	4.376095	4.637950	4.675745	4.827210
50	0.854640	0.915090	0.924740	0.959700	4.143885	4.499440	4.559055	4.759335
55	0.800330	0.883290	0.897310	0.942830	3.830325	4.315295	4.389845	4.651825
60	0.728840	0.840460	0.855860	0.915900	3.428780	4.058430	4.134605	4.481505
65	0.640480	0.779960	0.794190	0.873850	2.959500	3.710160	3.763960	4.233190
70	0.540820	0.700400	0.706570	0.811630	2.411275	3.245505	3.249725	3.848085
75	0.421010	0.593330	0.588740	0.722540	1.784990	2.646445	2.593935	3.316440
80	0.293170	0.462100	0.445210	0.597920	1.160305	1.946530	1.819305	2.589670
85	0.173540	0.314100	0.281000	0.431120	0.624270	1.202460	1.019395	1.685395
90	0.082120	0.171120	0.133350	0.244390	0.263825	0.579340	0.412640	0.825485
95	0.029410	0.068920	0.042170	0.096380	0.083085	0.199285	0.107340	0.267860
100	0.007540	0.018310	0.007730	0.022440	0.021720	0.049360	0.017030	0.053510
					68.185623	74.931633	74.786475	79.979545

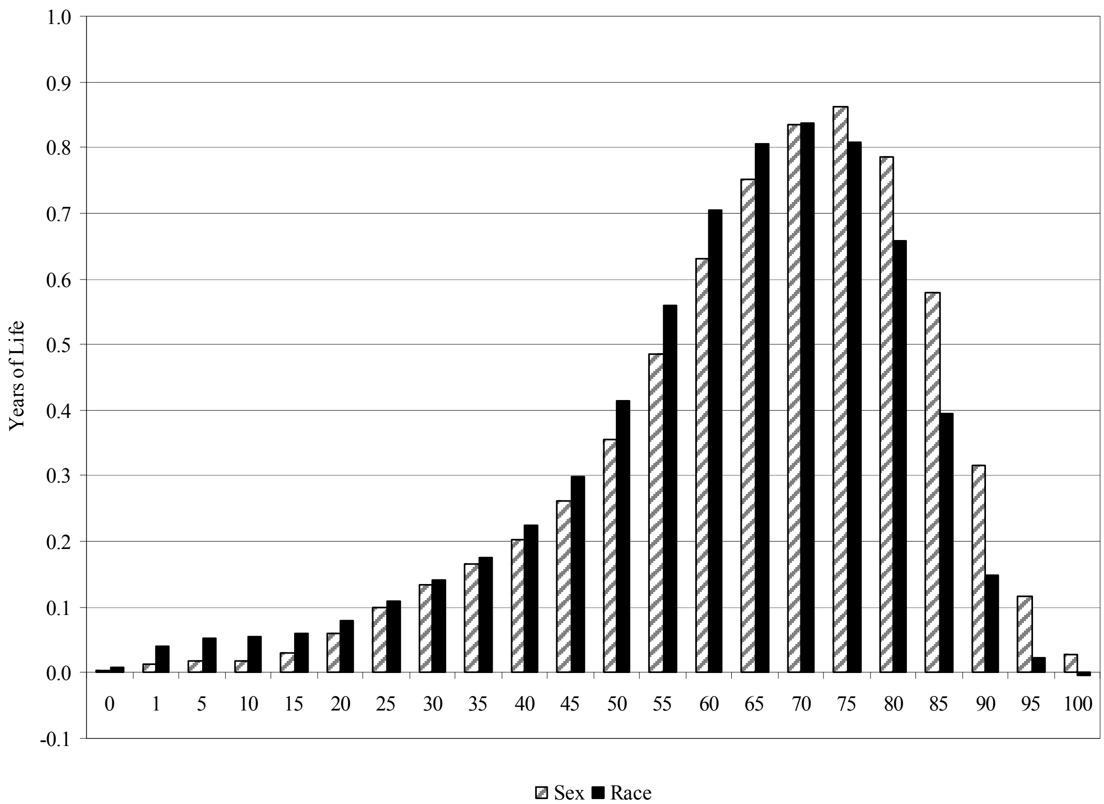
Source: Source: abridged from official complete U.S. Life Tables by Single Years of Age, National Center for Health Statistics.

Figure 1 - Age Origin of Black Male Deficit in Life Expectancy
by Race or Sex



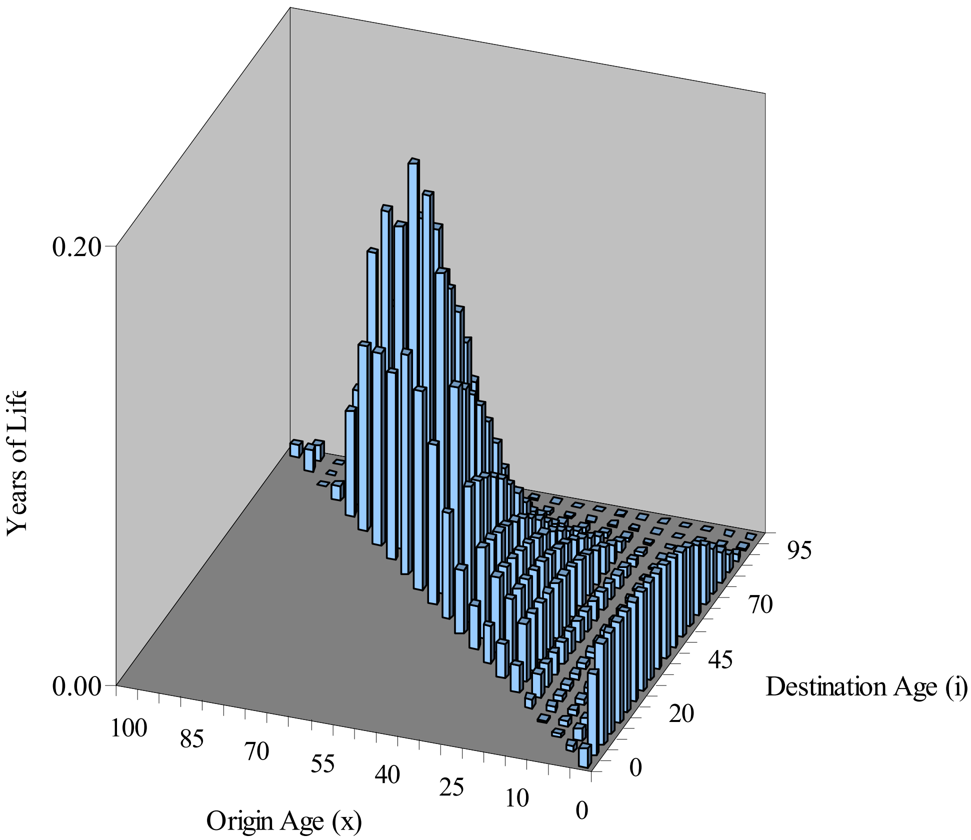
Source: data in Table 1.

Figure 2 - Destination Ages for Black Male Deficit in Life Expectancy
by Race or Sex



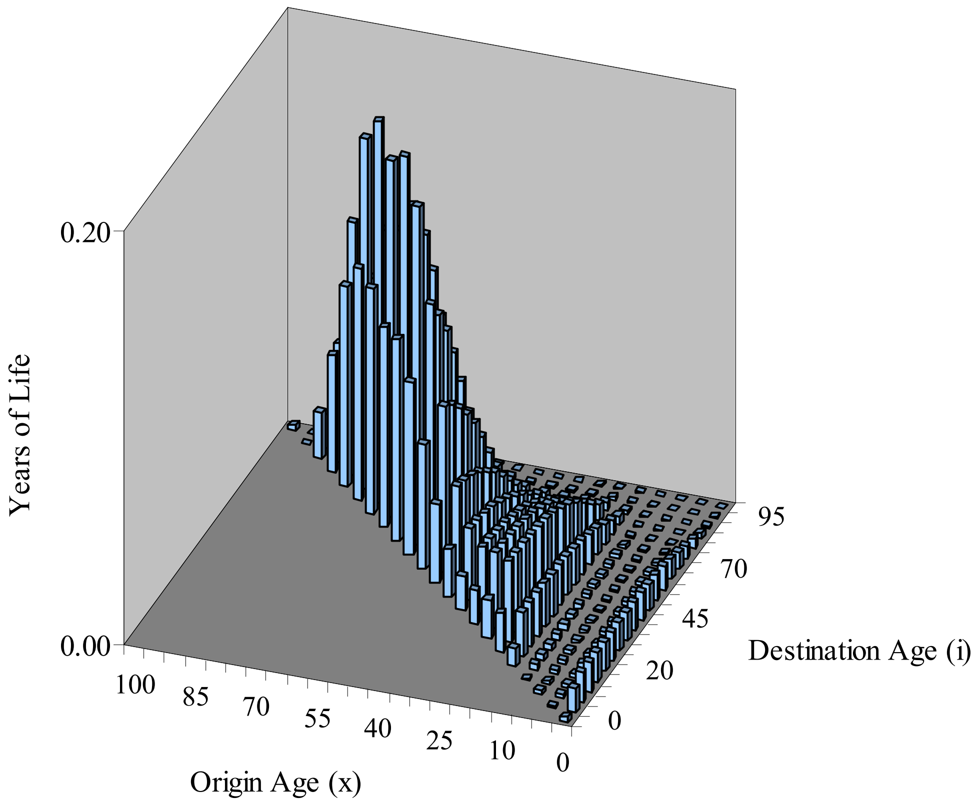
Source: data in Table 1.

Figure 3 -
Origin & Destination of Race Difference in Life Expectancy
(U.S. White Men-Black Men)



Source: data in Table 1.

Figure 4 -
Origin & Destination of Sex Difference in Life Expectancy
(U.S. Black Women-Black Men)



Source: data in Table 1.