Where is Poverty Greatest in Canada?
Comparing Regional Poverty Profile Without Poverty Lines:
A Stochastic Dominance Approach

Wen-Hao Chen[1]

Family and Labour Studies
Statistics Canada

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1. Introduction

Regional income disparity has long been a phenomenon in Canada. Partly, it is due to the cost-of-living differences across characterized regions within the country. It may also be the differences to human-capital-related demographic characteristics. Examining the extent in magnitude of differences among regional income distributions, particularly poverty outcomes often has important policy implications, as it is the basis for evaluating existing provincial welfare policies and for developing redistributive policies of the Federal government of Canada, such as the fiscal equalization grants.[2] In Canada, a regional poverty profile is not officially conducted but can often be obtained from constructing poverty measures (e.g., headcount ratio or income gap) based on Statistics Canada's Low-Income Cutoffs (LICOs). However, questions have been raised concerning how to obtain a reliable and robust regional profile, particularly when the measurement of welfare function and poverty is an issue of great controversy. It can be argued that any changes to method of defining poverty line or poverty measure could possibly lead to a very different geographic distribution of poverty.

This paper provides a robust way to compare regional poverty profile in Canada without arbitrarily specifying a poverty line. The analysis is carried out by an application of stochastic

[1] Correspondance: wen-hao.chen@statcan.ca; voice: 613-951-3807.
[2] Note that the Canadian fiscal equalization policy, which was entrenched as in early 1980s, is the system of grants from the federal government to the provincial and territorial governments.

dominance, which examines whether income distribution in one province has unambiguously more or less poverty than in another over ranges of potential poverty lines as well as over general classes of poverty indices. Robustness of the profile is also verified with respect to different choice of spatial price deflators, and equivalence scales. In addition, we also examine the extent to which the findings are sensitive to the choice of an absolute or a relative concept of poverty. Statistical inferences for stochastic dominance are established to account for sampling variations.

2. Research Methods

For poverty measures, we follow the Foster-Greer-Thorbeck (FGT) class of poverty indices (see Foster *et al*., 1984), which may be written generally as:

$$P_\alpha = \frac{1}{N}\sum_{i=1}^{N}(\frac{g_i}{z})^\alpha \ , (\alpha \geq 0);\ g_i = \max(z - y_i, 0)$$

where y_i is the value of needs-adjusted equivalent income for the *i*-th person, and g_i is the income short-fall for individual *i* for a given poverty line *z*, and α is a measure of the sensitivity of the index to poverty. Poverty comparisons between two income distributions are addressed by the technique of stochastic dominance, which is based on the comparisons of cumulative distribution functions (CDF). Consider two distributions of incomes with CDF, F_A and F_B respectively. Let

$D^1(x) = F(x)$, and

$D^s(x) = \int_0^x D^{(s-1)}(y)dy$ for any integer $s \geq 2$.

Distribution *B* is said to dominate distribution *A* stochastically at first order if the poverty incidence curve for *B* lies *nowhere* above that for *A*, $F_A(x) \geq F_B(x)$, up to arbitrarily selected maximum poverty line z_{max}. Similarly, second- and third-order dominance of *A* by *B* up to z_{max},

requires $D_A^2(x) \geq D_B^2(x)$ and $D_A^3(x) \geq D_B^3(x)$, respectively, for all $x \leq z_{max}$. Since the samples for the different provinces are independent, asymptotic variance estimates for the differences $\hat{D}_A^s(x) - \hat{D}_B^s$ are obtained by adding the variance estimates for distributions *A* and *B*:

$Var(\hat{D}_A^s(x) - \hat{D}_B^s(x)) = Var(\hat{D}_A^s(x)) + Var(\hat{D}_B^s(x))$.

To test whether one distribution dominates the other, we follow the approach of Bishop *et al.* (1991) that compare *t* tests for a set of sample ordinates, $\hat{D}_A^s(x) - \hat{D}_B^s(x)$. The dominance results hold if there is at least one positive significant difference and no negative significant differences between ordinates. Two distributions are ranked as equivalent if there are no significant differences. The curves cross if the difference in at least one set of ordinates is positive and significant while at least one other set is negative and significant.

3. Data and Definitions

The empirical analysis is carried out for ten provinces using the Survey of Labour and Income Dynamics for 2000. Sample includes everybody in the survey, and family is defined as economic family. Incomes are needs-adjusted as well as cost-of-living adjusted based on the conversion factors that used for the calculation of the LICOs. To examine the robustness of the dominance results for different definitions of needs-adjusted incomes, we redo the same exercise using two alternative equivalence scales (squared root of family size and modified OECD scales) and another set of cost-of-living deflator that used for the calculation of the Market Basket Measure (MBM) to make income adjustment. The robustness analysis is further extended to include tests of dominance using purely relative poverty lines, as determined by a proportion of median income, since whether or not poverty should be regarded as an absolute or a relative concept is an issue of great debate.

4. Expectations

Canada has historically relied on the LICOs to address issues of low income or poverty. However, criticisms have always been raised for its arbitrary selection with respect to proportion spending on necessities and what constitute necessities. It is argued that any revision of these standards would lead to completely different poverty outcomes. This paper will overcome such shortcomings and provides poverty comparisons that are robust to the choice of poverty lines. This paper will also provide evidence on how sensitive is dominance relation to the assumptions made to costs of living differences as well as to poverty concept. Specifically, we are expecting the following results.

1. Poverty ordering among provinces can be established based on tests of dominance relation, up to third order condition.
2. Dominance relation disappeared or a rank-reversal occurred when a different choice of cost-of-living deflator or a relative poverty concept is applied.
3. For all scales, dominance results tend to be more robust for some provinces (likely British Columbia) but less so for the other provinces (likely Ontario and Quebec).

References

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