Health, Survival and Consumption over the Life Cycle: An Optimal Control Approach*†

Michael Kuhn‡, Alexia Prskawetz§, Stefan Wrzaczek¶, Gustav Feichtinger¶

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Abstract

In recent years most industrialized countries have allocated increasing shares of their GDP to health while at the same time life-expectancy has continued to increase. The ongoing debate in health economics is on whether too much is spent on health care. In this paper we offer a framework that allows to determine how a social planner as opposed to an individual allocates resources to consumption vs. the provision of health care over the life cycle assuming that health care positively affects longevity. Applying the concept of the willingness to pay for mortality reductions, we derive the social versus private value of life.

1 Introduction

In recent years most industrialized countries have allocated increasing shares of their GDP to health. In the US health expenditures share in GDP increased from 5 % in 1970 to 15-16 % in 2000. Similar increases can be observed in Germany and Japan where health expenditure's share in GDP increased from 6 % in 1970 to 11 % in 2000

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‡Max Planck Institute for Demographic Research, Konrad-Zuse-Str. 1, D-18057 Rostock, Germany (kuhn@demogr.mpg.de)

§Vienna Institute of Demography, Prinz Eugenstr. 8-10, A-1040 Vienna, Austria (alexia.fuernkranz-prskawetz@oeaw.ac.at)

¶Institute of Mathematical Methods in Economics (Research group on Operations Research and Non-linear Dynamical Systems), Vienna University of Technology, Argentinierst. 8, A-1040 Vienna, Austria (wrzaczek@server.eos.tuwien.ac.at)

for Germany and from 3 % in 1950 to 7-8 % in 2000 for Japan (see Bergheim [1]). At the same time life-expectancy has continued to increase. Whereas increasing life-expectancy is more than welcome as such, a debate is continuing on whether or not too much is being spent on health care. This begs two questions: (i) what motivates individuals to invest in reductions in mortality and to what effect? (ii) Do they get it right from a social welfare point of view? We seek to provide an answer to these questions by combining two models:

1. an age structured optimal control model, where a social planner maximizes welfare (i.e. individual utilities aggregated over time and age groups). This model determines the socially optimal pattern of consumption and health investments.

2. a life-cycle model, where an individual maximizes life-time utility. This model determines the individual pattern of consumption and health investment.

Solving and simulating models (1) and (2) and comparing the respective patterns of consumption and health investment we can deduce conclusions about the inefficiencies in individual behavior and where they arise.

Our model of individual behaviour (model (2)) is closely related to the work of Ehrlich and Chuma [4], Ehrlich [3] and Hall and Jones [9]. As argued in Ehrlich and Chuma [4] (p. 762) "the observed diversity in age specific life expectancy over time and across different population groups "may be due not just to the influence of exogenous biological or technological factors but also to systematic variations in individuals' demand for longevity." To determine the demand for longevity an intertemporal setting is needed where the demand function for longevity can be modelled along with the demand function for health investment and consumption. The seminal work on the demand for health is Grossman [8] who models longevity as the outcome of health capital that in turn is produced by health investments.

Similar to Ehrlich [3] the individuals maximize the discounted stream of utility obtained from consumption over their life cycle by choosing how much to spend on health care and consumption subject to their individual budget constraint. We ignore quality of life that also enters the objective function in Ehrlich [3]. Ehrlich and Chuma [4] assume that the stock of health has two functions: firstly, it augments the amount of healthy time and secondly, it delays the approach of death since the latter is assumed to occur automatically once the health stock drops to a critical minimum level. The health stock can be maintained or augmented by investment and depreciates over time. In our model we ignore the first channel and only concentrate on the reduction in mortality through health investment. Similar to Ehrlich [3] we assume that health care affects mortality.

In our model 1, we consider a social planner that maximizes the discounted stream of utility of the population by choosing age specific health investment and consumption subject to an economy-wide budget constraint. In this case we need to model the evolution of the total population. We refer to the McKendrick equation in demography to model the evolution of population size over time and age. Similar to the individual optimisation problem, health spending enters negatively the age specific mortality function. Moreover, in the social planner model we introduce age in addition to time as a second dynamic variable. Health care spending is therefore age *and* time specific in our model.

Within our model framework we can then determine how a social planner as opposed to an individual allocates resources to consumption vs. health investment over the life cycle assuming that health investment positively affects longevity. In particular, by introducing age as a second dynamic variable in addition to time, we are able to study the age patterns of the optimal health investment and consumption paths as endogenously determined by the population structure and age specific productivity (we assume that output is produced by labor only where workers of different ages are adjusted for their different productivity). Differences between the planners and individual's investment/consumption patterns indicate a potential need of policy intervention.

By applying the optimality conditions of age structured optimal control models (Feichtinger, Tragler and Veliov [6]) we can derive the social optimal age specific consumption and health investment profile. The social optimum is determined by equalizing the marginal consumption to the marginal social benefit of an increase in the population by one member through a reduction in mortality. A measure commonly applied in the life-cycle models (also in connection with health investments) is the willingness to pay for a small reduction for mortality (Rosen [14]), termed the Value of Life (VOL). Our framework allows us to derive the Social VOL (SVOL) as a function of the value of one more individual at age a and time t multiplied by the number of people and divided by the marginal utility of consumption to obtain a monetary value of SVOL. We also offer an intuitive explanation of the value of one more individual at age a and time t and show its close connection to the reproductive value concept in demography.

For the individual optimum we can derive similar results. In the optimum the marginal utility of consumption is set equal to the marginal private benefit of health, i.e. the benefit from a decrease in mortality. Similar to the SVOL we can derive a PVOL (Private value of life) that indicates the willingness to pay for an additional life year. The PVOL turns out to be equal to the marginal costs (in terms of foregone consumption) of a life year.

2 The Social Welfare Model

The dynamics of the population is described by the McKendrick equation (see Keyfitz [12])

$$N_a + N_t = -\mu(a, h(a,t))N(a,t) \qquad\qquad N(0,t) = B(t), N(a,0) = N_0(a) \tag{1}$$

The state variable $N(a,t)$ represents the number of a-year old individuals at time t. The age specific mortality rate $\mu(a, h(a,t))$ trivially depends on age a and can be reduced instantaneously by providing to the individual an age specific amount $h(a,t)$ of health care (or other health enhancing goods and services). Here, $h(a,t)$ is a distributed control variable in our model. We model the mortality rate according to the proportional hazard model (see Kalbfleisch and Prentice [11])

$$\mu(a, h(t,a)) = \tilde{\mu}(a)\phi(h(a,t)) \tag{2}$$

where $\tilde{\mu}(a)$ denotes the base mortality rate (effective in the absence of any health care) and $\phi(h(a,t))$ describes the impact (efficiency) of health spending. We assume that $\phi(h(a,t))$ is a strictly decreasing concave function satisfying the Inada conditions, i.e. $\phi_h < 0$, $\phi_{hh} > 0$, $\phi(0) = 1$ and $\phi_h(0) = -\infty$[1]. Note, that the proportional hazard model implies that the effectiveness of health care in reducing mortality increases in the hazard rate of mortality.

$N_0(a)$ describes the initial age distribution of the population and $B(t)$ equals the number of newborns at time t defined as

$$B(t) = \int_0^\omega \nu(a)N(a,t)\ da \tag{3}$$

where $\nu(a)$ denotes the age specific fertility rate.

The second control variable is consumption $c(a,t)$. The nonnegativity assumption is trivially fulfilled, as we assume $\lim_{c\to 0+} u_c(c) = +\infty$ for the utility function. The objective of the social planner is to maximize the social welfare, which is defined as the sum of the instantaneous utilities of all individuals (total utilitarianism)

$$\int_0^T \int_0^\omega e^{-\rho t} u(c(a,t))N(a,t)\ da\ dt \tag{4}$$

where ω is the maximal age an individual can reach. This is no restriction to the model if ω is chosen big enough. $u(c(a,t))$ represents the per capita instantaneous utility, which depends only on consumption and is assumed to be concave in its argument. In some life-cycle models utility might also depend on health (quality of life). However, in our approach health care only influence the mortality (e.g.

[1]Thus the usual assumption of nonnegative health investments is not necessary.

vaccination) and not the quality of life (e.g. glasses or contact lenses) or even both (e.g. medicines that reduce blood pressure). ρ denotes the (social) rate of time preference. Note that we also allow for an infinite planning horizon $T = +\infty$ at this stage.

Finally we assume that the budget condition needs to be balanced over the planning horizon T. This is expressed by the introduction of savings (of the economy) $S(t)$ and the following dynamics

$$\dot{S}(t) = rS(t) + Y(t) - C(t) - H(t) \qquad\qquad S(0) = S(T) = 0 \tag{5}$$

with

$$\begin{aligned} Y(t) &= \int_0^{\omega} p(a)N(a,t)\ \ da \\ C(t) &= \int_0^{\omega} c(a,t)N(a,t)\ \ da \\ H(t) &= \int_0^{\omega} h(a,t)N(a,t)\ \ da \end{aligned} \tag{6}$$

r denotes the interest rate that is exogenous to the economy, $p(a)$ denotes the productivity of an a-year old individual and $Y(t)$ and $C(t)$ total output and consumption, respectively. We assume that health care is purchased at a relative price, which we normalize to one. Thus, $h(t)$ and $H(t)$, respectively, correspond to per capita and aggregate health expenditure. The economy starts and ends up with zero savings.

The formal problem of the social planner is then to choose the age specific schedule of consumption and health expenditure (health care) to maximize the sum of instantaneous utility of all individuals. Discounting the future at the rate ρ we come up with the following dynamic age-structured optimization problem with state variables $S(t)$ and $N(t)$ and control variables $c(a,t)$ and $h(a,t)$.

$$
\begin{aligned}
\max_{c,h} \quad & \int_0^T \int_0^\omega e^{-\rho t} u(c(a,t)) N(a,t) \ da \ dt \\
\text{s.t.} \quad & N_a + N_t = -\mu(a, h(a,t)) N(a,t) \\
& \quad N(0,t) = B(t) = \int_0^\omega \nu(a) N(a,t) \ da \\
& \quad N(a,0) = N_0(a) \\
& \quad \mu(a, h(a,t)) = \tilde{\mu}(a) \phi(h(a,t)) \\
& Y(t) = \int_0^\omega p(a) N(a,t) \ da \\
& C(t) = \int_0^\omega c(a,t) N(a,t) \ da \\
& H(t) = \int_0^\omega h(a,t) N(a,t) \ da \\
\text{budget:} \quad & \dot{S}(t) = rS(t) + Y(t) - C(t) - H(t) \\
& \quad S(0) = S(T) = 0
\end{aligned} \tag{7}
$$

In the following section we derive the necessary optimality conditions of the above age specific control problem (see Feichtinger, Tragler and Veliov [6] for details). Further we provide economic interpretations of important expressions.

3 Optimality conditions and the social value of life

To obtain necessary optimality conditions we apply the maximum principle for age-structured control models as recently derived in [6].

We define the Hamiltonian of the social welfare problem as follows: (from now on we omit a and t if they are not of particular importance)

$$\mathcal{H} = u(c)N - \xi^N \mu(a,h)N + \xi^S (rS + Y - C - H) + \eta^B \nu N + \eta^Y pN + \eta^C cN + \eta^H hN \tag{8}$$

where we denote the adjoint variables that correspond to the state variables as follows:

- $\xi^N(a,t)$ ... population
- $\xi^S(t)$ ... savings
- $\eta^B(t)$ ... newborns

- $\eta^Y(t)$ ... output
- $\eta^C(t)$ ... total consumption
- $\eta^H(t)$ ... total health expenditure

such that the following system is satisfied

$$\begin{aligned}
\xi^N_a + \xi^N_a &= (\rho + \mu(a,h))\xi^N - u(c) - \eta^B \nu - \eta^Y p - \eta^C c - \eta^H h \\
\dot{\xi}^S &= (\rho - r)\xi^S \\
\eta^B &= \xi^N(0,t) \\
\eta^Y &= \xi^S \\
\eta^C &= -\xi^S \\
\eta^H &= -\xi^S
\end{aligned} \tag{9}$$

together with

$$\xi^N(\omega, t) = 0 \tag{10}$$

If $T < +\infty$ we further have $\xi^N(a,T) = 0$. The necessary first order conditions are

$$\mathcal{H}_c = u_c(c)N + \eta^C N = 0 \tag{11}$$
$$\mathcal{H}_h = -\xi^N \mu_h(a,h)N + \eta^H N = 0 \tag{12}$$

After combining them we obtain

$$u_c(c)N = -\xi^N \mu_h(a,h)N \tag{13}$$

This equation can be interpreted in a straight forward manner. The left hand side equals the increase of social welfare if $c(a,t)$ is increased by a small (marginal) unit. The right hand side is the product of the adjoint variable of the population ξ^N and $\mu_h(a,h)$. As usual ξ^N can be interpreted as shadow price and is therefore equal to the increase of the value function (i.e. social welfare) for a small (marginal) increase of $N(a,t)$. $-\mu_h(a,h)N$ equals the value of the number of lives saved through a marginal increase of $h(a,t)$. Therefore the right hand side represents the increase of social welfare if health expenditure is increased by a small (marginal) unit. In an optimum both sides must be equal. Or, writing $u_c(c) = -\xi^N \mu_h(a,h)$, the marginal utility of consumption equals the marginal value per-capita of a health care spending.

From (9) and (11) we get $u_c(c) = \xi^S$. This equation implies that the consumption at each point in time t is equal over all ages a, as the shadow price of savings only depends on time t and not on age a.

The change in consumption of a cohort born at $t-a$ can be expressed by the following formula (obtained by calculating the directional derivative of (13))

$$c_a + c_t = \frac{u_c(c)}{u_{cc}(c)}(\rho - r) \tag{14}$$

If the discount rate equals the international market interest rate the above formula is zero implying consumption smoothing over the whole life. If the discount rate exceeds the international market interest rate the formula is negative, because of the concavity of the utility function. Thus the consumption will decrease over the life-cycle, which reflects the impatience of the individuals is greater than the interest rate. In the case of $r > \rho$ the interpretation is the other way around, i.e. consumption increases over the life cycle.

Therefore in general the consumption will not be smoothed over all periods and all ages, although this would be possible for the social planner. All in all it can be concluded that consumption does not differ across age groups at one point in time, but increases ($\rho < r$), decreases ($\rho > r$) or stays constant ($\rho = r$) over the entire planning horizon (and the life-span of every single individual).

Finally we calculate the willingness to pay for a small reduction of the mortality rate for age a at time t. To our knowledge this concept was firstly dealt with in a formal manner in Rosen [14] who applied the *value of life* (VOL) concept to a single individual in a life cycle model. As our approach uses a macro economic setting we term it consequently *social value of life (SVOL)*. Before we discuss the differences between the VOL and the SVOL we derive an analytic expression for the SVOL. Analogously to Rosen [14] the SVOL, denoted by $\Psi^S(a,t)$, equals

$$\Psi^S(a,t) = -\frac{\partial V/\partial \mu}{\partial V/\partial S} \tag{15}$$

where V denotes the value function, i.e. optimal social value. This formula expresses the marginal rate of substitution (MRS) between mortality and social wealth, which describes the slope of the indifference curve including all combinations of $\mu(a, h(a,t))$ and $S(t)$ with the same social welfare. The denominator equals the shadow price of savings ξ^S. For the numerator we obtain

$$\frac{\partial V}{\partial \mu} = \frac{\partial V}{\partial N}\frac{\partial N}{\partial \mu} = -\xi^N(a)N(a) \tag{16}$$

The second equality can straightforwardly be verified by solving the partial differential equation for N by the method of characteristics. Putting the above two expressions together we obtain

$$\Psi^S(a,t) = \frac{\xi^N(a,t)N(a,t)}{\xi^s} = \frac{\xi^N(a,t)N(a,t)}{u_c(c)} \tag{17}$$

The value ξ^N of one more (more precisely: marginally more) a-year old individual at time t is multiplied by the number of them N, as only such individuals will benefit from a reduction of the mortality rate in this specific age at that time. This value is devided by the marginal utility, such that the SVOL is expressed in monetary values.

The change of the SVOL within one cohort born at time $t-a$ is given by

$$\Psi_a^S + \Psi_t^S = r\Psi^S - N\Big(\frac{u(c)+\eta^B\nu}{u_c(c)} + (p-c-h)\Big) \tag{18}$$

Both terms are similar in structure to the expression in equation (34) (first line) in Rosen [14] (or, similarly, to the equation just above equation (10) in Murphy and Topel [13]). The SVOL for this particular cohort tends to increase (along the Lexis Diagram) if discounted future surplus (the first term) exceeds the current social surplus of this cohort (the second term). Here, the current social surplus amounts to the per-capita surplus (monetary value of utility + reproductive value + net productive value) multiplied by the number of cohort members alife in year t.

4 Analogy of the shadow price of population to the reproductive value

In this section we discuss an interesting interpretation of the shadow price of the population $\xi^N(a,t)$ from a demographic point of view. Remember the first equation of (9) representing the evolution of $\xi^N(a,t)$ for one cohort born at $t-a$ (45 degree line in the Lexis diagram). Together with the terminal condition $\xi^N(\omega,t)=0$ this partial differential equation can be solved with the method of characteristics. We obtain

$$\xi^N(a,t) = \int_a^\omega \big(u(c) + \xi^N(0,t-a+s)\nu(s) + \xi^S(p-c-h)\big)e^{-\rho(s-a)-\int_a^s \mu(a,h)\ ds'}\ ds \tag{19}$$

This expression can be decomposed into two effects, a direct and an indirect one, i.e.

$$\begin{aligned}\xi^N(a,t) &= \int_a^\omega e^{-\rho(s-a)}\big(u(c)+\xi^S(p-c-h)\big)e^{-\int_a^s \mu(a,h)\ ds'}\ ds + \\ &\quad + \int_a^\omega e^{-\rho(s-a)}\xi^N(0,t-a+s)\nu(s)e^{-\int_a^s \mu(a,h)\ ds'}\ ds \end{aligned} \tag{20}$$

The direct effect (first integral) is the sum of the instantaneous utility of consumption $u(c)$ (at age s and time $t-a+s$) and the net contribution to social welfare by one individual $\xi^S(p-c-h)$ (if $p-c-h>0$: the individual adds to social welfare as its contribution to production outweighs its consumption and health investment, and vice versa for $p-c-h<0$) both weighted by the discount factor and the survival probability and aggregated over the remaining lifespan of one individual. Therefore the direct effect describes the additional social welfare (for one individual itself and for others) that is generated by preserving the life of one individual.

The indirect effect (second integral) equals the shadow price of a newborn individual at time $t-a+s$ times the fertility rate of an s-year old individual again weighted by the discount factor and the survival probability and aggregated over the remaining lifespan. Consequently it measures the expected social welfare generated by the descendants of an additional individual of age a at time t.

Interestingly the indirect effect is a generalization of the reproductive value $v(a)$ (see Fisher [7]), which is a well known concept in demography and defined in the following way[2]

$$v(a) = \int_a^\beta e^{-r(s-a)} \frac{l(s)}{l(a)} m(s) \; ds \tag{21}$$

Here $l(s)$ denotes the probability to survive until age s, thus $\frac{l(s)}{l(a)}$ denotes the survival probability between ages a and s and is equivalent to the term $e^{-\int_a^s \mu(a,h)\; ds'}$ in our indirect effect. $m(s)$ denotes the fertility rate of age s and equals trivially our $\nu(s)$. As β denotes the oldest age of child bearing the only difference in both expressions is the shadow price of one additional newborn, which does not occur in (21). This difference arises from the fact that (21) measures the reproductive value in amounts of individuals, i.e. one individual has value 1. Our indirect effect is expressed in additional units of utility and thus multiplied by $\xi^N(0, t-a+s)$. Consequently it measures the expected social welfare generated by the descendants of an additional individual. Now it is obvious that we have obtained a generalization, when $\xi^N(0, t-a+s)$ denotes the value of an additional newborn depending on the measure (e.g. number of individuals in the classical representation and units of utility in our model).

5 Individual Choice Model

In this section we briefly introduce an individual choice model, as we aim at a comparison between the social optimal age specific consumption and health care spending and that of an individual choice model.

Each individual earns wage $w(a)$, which is assumed to be equal to age specific productivity $p(a)$. Savings (again with interest rate r) over time are allowed as long

[2]Note that in this formula we use demographic notation.

as they are nonnegative. This reflects a scenario in which individuals cannot pool their mortality risks, e.g. through the purchase of life insurance. Credit market institutions will then usually not allow individuals to die with a negative net wealth (Yaari [16], Ehrlich [3]). Hence, individual wealth develops according to

$$\dot{S}(a) = rS(a) + w(a) - c(a) - h(a) \qquad\qquad S(0) = 0, S(a) \geq 0. \tag{22}$$

Disregarding planned-for bequests, we obtain $S(\omega) = 0$ The probability of surviving to age a (modelled analogously to the social planner problem) equals

$$\exp\big(-\int_0^a \mu(s,h)\ ds\big) \tag{23}$$

with $\mu(a, h(a)) = \tilde{\mu}(a)\phi(h(a))$, where all functions and variables have the same meaning as in the social welfare model, but here only evolving over age. We further assume $u_c(c) > 0$ and $\lim_{c\to 0+} u_c(c) = +\infty$.

The individual then maximizes utility by choice of consumption and the procurement of health care according to

$$\begin{aligned}
\max_{c,h} \qquad & \int_0^\omega e^{-\rho a} u(c(a)) e^{-\int_0^a \mu(s,h(s))\ ds}\ da \\
\text{s.t.} \quad & \dot{S}(a) = rS(a) + w(a) - c(a) - h(a) \qquad S(0) = 0, S(a) \geq 0, S(\omega) = 0 \\
& \mu(a,h) = \tilde{\mu}(a)\phi(h(a))
\end{aligned} \tag{24}$$

Note that the restriction $c(a) \geq 0$ and $h(a) \geq 0$ is again not necessary, because of the assumption concerning the shape of $u(c)$ and $\phi(h)$.

The survival probability can also be formulated as state $M(a)$ with initial condition $M(0) = 1$. Thus the control problem can also be formulated as

$$\begin{aligned}
\max_{c,h} \qquad & \int_0^\omega e^{-\rho a} u(c(a)) M(a)\ da \\
\text{s.t.} \quad & \dot{M}(a) = -\mu(a, h(a)) M(a) \qquad\qquad M(0) = 1 \\
& \dot{S}(a) = rS(a) + w(a) - c(a) - h(a) \qquad S(0) = 0, S(a) \geq 0, S(\omega) = 0 \\
& \mu(a,h) = \tilde{\mu}(a)\phi(h(a))
\end{aligned} \tag{25}$$

The Lagrangian of this problem reads (again omitting a if it is not of particular importance)

$$\mathcal{L} = u(c)M - \lambda_M \mu(a,h) M + \lambda_S(rS + w - c - h) + \lambda S \tag{26}$$

where λ_M and λ_S are the adjoint variables of the survival probability and individual savings respectively. λ denotes the Lagrangian multiplier of the nonnegativity constraint of $S(a)$.

From the necessary optimality conditions we can derive the following system of adjoint variables:

$$\begin{aligned}\dot{\lambda}_M &= (\rho + \mu(a,h))\lambda_M - u(c) \\ \dot{\lambda}_S &= (\rho - r)\lambda_S - \lambda\end{aligned} \tag{27}$$

together with the first order condition:

$$\begin{aligned}\mathcal{L}_c &= u_c(c)M - \lambda_S = 0 \\ \mathcal{L}_h &= -\lambda_M M \mu_h(a,h) - \lambda_S = 0\end{aligned} \tag{28}$$

We again combine them and obtain

$$u_c(c) = -\lambda_M M \mu_h(a,h) \tag{29}$$

where λ_M denotes the shadow price of survival. The interpretation is analogous to that of the social welfare model. The change in consumption over the life cycle is given by

$$\dot{c} = \frac{u_c(c)}{u_{cc}(c)}(\rho - r + \mu(a,h)) - \frac{\lambda}{M u_{cc}(c)} \tag{30}$$

where λ is the Lagrangian multiplier of the constraint $S(a) \geq 0$. According to the complementary slackness condition λ is zero, whenever $S(a) > 0$, i.e. if $S(a)$ is not at the boundary. For the interpretation consider $\lambda = 0$. Then the consumption path is equal to that of the social welfare problem for one cohort except that the mortality rate is additionally added in the bracket. So if $\rho > r$ consumption will decrease over the life-cycle even faster than in the social welfare problem because of the mortality rate. If $\rho = r$ consumption will decrease. And if $\rho < r$ the effect is not clear. For a very low mortality rate (e.g. in younger years) consumption will still increase (as in the social welfare model). But when mortality is high enough (to fill the gap between ρ and r) consumption will decease in this case.

Thus the above expression includes two effects. Firstly $\rho - r$ describes, whether the impatience is greater than the market interest rate. And secondly the mortality effect, which can outweight high patience or increase impatience. The mortality rate therefore counts for the effect of a decreasing survival probability, as an individual cannot be sure to be able to consume its savings (which is the main difference to the social welfare problem).

If $S(a)$ is on the boundary $\lambda \geq 0$ (nonnegativity complementary slackness condition), which implies that the last term is negative. Thus the consumption path decreases more or increases less (depending on the first term), as it would without the nonnegativity constraint.

The private value of life (PVOL) (calculated according to the approach of Rosen [14]), denoted by Ψ^P, equals[3]

$$\Psi^P = \frac{\lambda_M}{u_c(c)} \tag{31}$$

The private value of life or, equivalently, the value of an additional life-year, amounts to the monetary value of the discounted stream of future utility. Using the first-order conditions it is also readily verified that $\Psi^P = \frac{\lambda_M}{u_c(c)} = -\frac{}{\mu_h(a,h)}$. Hence, individuals choose the amount of health care that equalises the PVOL with the effective marginal cost of extending life. Note that this cost decreases in the effectiveness of health care (in reducing mortality), i.e. in $\mu_h(a, h)$.

Using the above, we obtain the change of the PVOL of one individual

$$\dot{\Psi}^P = r\Psi^P - \frac{u(c)}{u_c(c)} + \frac{\lambda}{u_c(c)M}\Psi^P \tag{32}$$

Considering the first two terms on the RHS, there is an obvious analogy to the SVOL. The PVOL increases if the discounted and monaterized stream of futrue utilily exceeds the present monetary value of utility and vice versa. Note, however, that in contrast to the social planner, individuals do not take into account neither their net productive value $p - c - h$ nor their reproductive value $\frac{\nu\eta^B}{u_c(c)}$. This reflects their pure selfishness. If the constraint on positive net wealth is binding, i.e. if $\lambda > 0$, this tends towards increasing PVOL over time, as individuals expect a greater scope for consumption in the future (when they are less credit constrained).

6 Numerical Results

For the numerical results we use the following functional specification

$$\begin{aligned} u(c(a,t)) &= b + \frac{c(a,t)^{1-\sigma}}{1-\sigma} \\ \phi(h(a,t)) &= 1 - \sqrt{\frac{h(a,t)}{z}} \end{aligned} \tag{33}$$

where $b = 5$, $\sigma = 2$ and $z = 3$. Further we choose $\rho = r = 0.04$ for the subjective discount rate and the market interest rate respectively. The simulation starts at age 19 and ends at the maximal life-span $\omega = 110$. We assume an exogenous number of births, i.e. $B(t) = B$. Consequently the indirect effect of the shadow price of population (introduced in section 4) is zero as $\eta^B = 0$. Mortality rates for

[3] Rosen [14] and several other papers use the term value of life. However, we use PVOL in order to distinguish it from the SVOL.

Germany have been taken from the human mortality data base [2] for the years 1990-2000. Further we separated the risk hump around age 20 from the natural mortality and assume that the mortality due to very risky behaviour in this age group cannot be influenced by health investments. Thus for the numerical calculations we use $\mu(a,h) = \tilde{\mu}(a)\phi(h) + \bar{\mu}(a)$, where $\tilde{\mu}(a)$ and $\bar{\mu}(a)$ are the base mortality and the risk hump[4] respectively. The age-specific productivity has been taken from Skirbekk [15], who calculates the weighted average over 6 age dependent abilities (numerical ability, managerial ability, clerical perception, finger dexterity, manual dexterity, experience). Consequently the productivity profile does not represent the productivity for a special profession, but the average over (more or less) all of them. The profile is plotted in figure 1. All our results are calculated in a stable population scenario with an exogenous constant number of births.

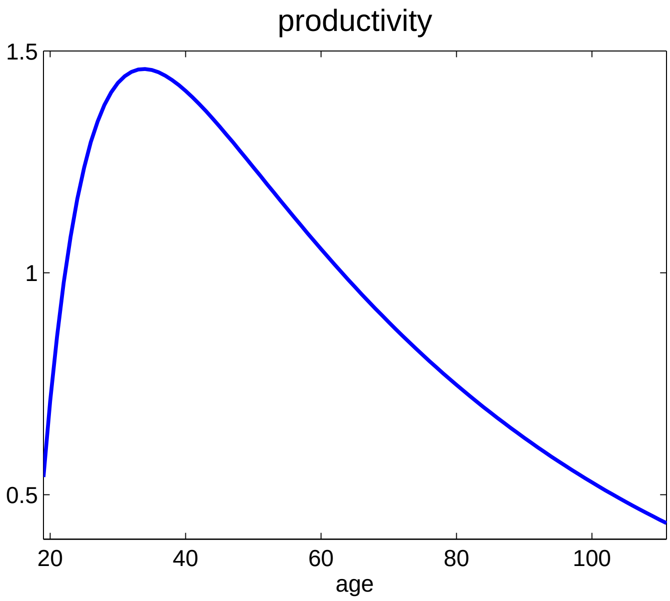

Figure 1: age-specific productivity profile

First we discuss the age-specific consumption and health investments. In the left panel of figure 2 we plot a comparison of the age-specific consumption of the social welfare model and the individual choice model. In addition we have included the age profile of productivity (which is equal to the wage of the individual choice model). The consumption in the social welfare is flat across age groups, reflecting the socially efficient distribution of consumption across time as we assume $\rho = r$. The consumption of the individual is hump shaped, it closely follows the productivity age

[4] $\bar{\mu}(a)$ is positive around age 20 and zero otherwise.

schedule in the young and old age ages, where individuals are credit constrained. In mid ages consumption is below the productivity schedule indicating positive savings. In this region the individual consumption profile is driven by the objective of consumption smoothing, which is modified, however, by the lack of insurance against unexpected mortality. Indeed, the decrease in consumption from ages 62 onwards is caused by the increasing probability of mortality against which individuals cannot insure themselves in our model.[5]

The comparison of health care expenditure is plotted in the right panel of figure 2. It is interesting to note that individual health care spending exceeds the social planner's spenidng at younger ages and falls short of the social optimum at higher ages. Over-spending on the part of individuals during their youth can be explained by their desire to ensure survival up to and during those ages (up to their early 80s) in which they enjoy high levels of consumption. The subsequent drop in individual health care expenditure as opposed to what would be socially optimal can be explained by two factors (relating to the PVOL as opposed to the SVOL): Firstly, in the absence of insurance they discount the PVOL for increasing risk of mortality. Secondly, the strongly reduced scope for consumption at old ages in the individual model (as opposed to the social optimum) drives down even more their value of surviving into the future.[6]

Finally we plot the PVOL (left) and the SVOL (right) in figure 3. The PVOL increases for the first years and decrease after a peak, which equals to the age at which consumption starts to decrease in Figure 2.

The SVOL is driven by two factors, i.e. the shadow price of population and the density of the population, which are both decreasing in the current setting. The marginal consumption is not crucial for the shape of the SVOL of one cohort, as it is constant across age. The interpretation of the rapid decrease of the SVOL is as follows. The SVOL measures the impact of changes in the mortality rate on the social welfare. One effect (making the decrease more fast) is that changes of the mortality rate are much more expensive for younger ages, as the base mortality rate is considerably lower. Another effect (making the decrease more slightly) is that a lower mortality rate increases social welfare as the surviving population will be larger (thus producing and consuming more). It is intuitive and obvious from the plot that the first effect dominates the second one.

[5]Adding actuarially fair life-assured annuities like in [10] would imply that the individual could smooth consumption.

[6]Note from $\mu_h(a,t) = \widetilde{\mu}(a,t)\,\phi_h(h)$ that the marginal effect on mortality of health investments increases with the baseline mortality $\widetilde{\mu}(a,t)$and therefore with age. For reasons of simplicity, we ignore in this model the age dependency of the $\phi-$technology, where one would expect a negative effect of age on ϕ_h at least for very high ages. We should stress, however, that while this effect would reduce health expenditure at the highest ages, this would affect both the individual and social pattern. The differences in expenditure which are of more material interest to us would remain mostly unaffected.

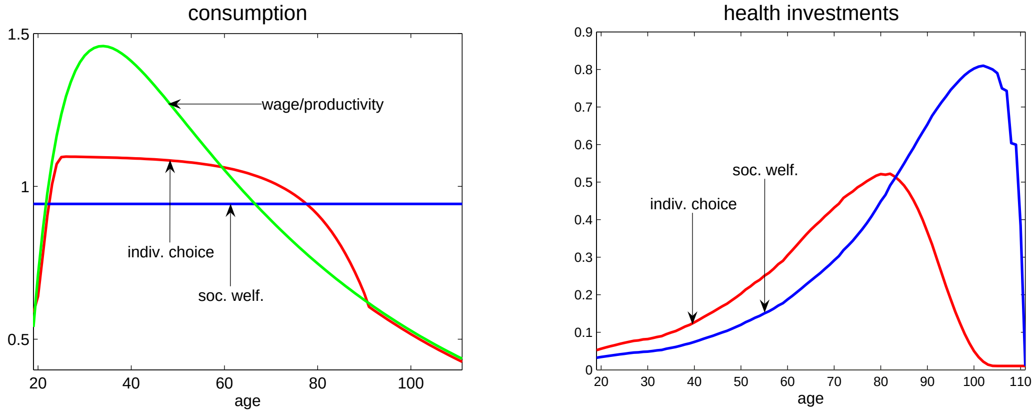

Figure 2: age-specific consumption (left) and health investments (right)

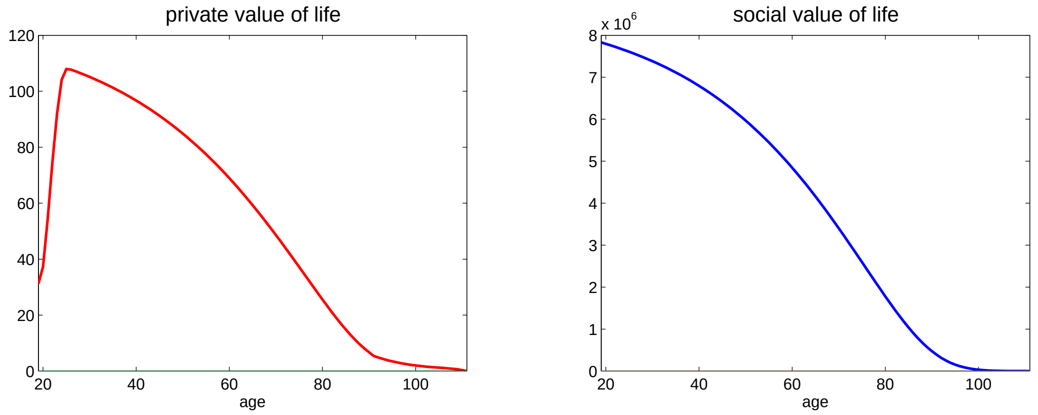

Figure 3: PVOL (left) and SVOL (right)

7 Conclusions

To be done.

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