A Re-evaluation of the Arriaga Fertility Method
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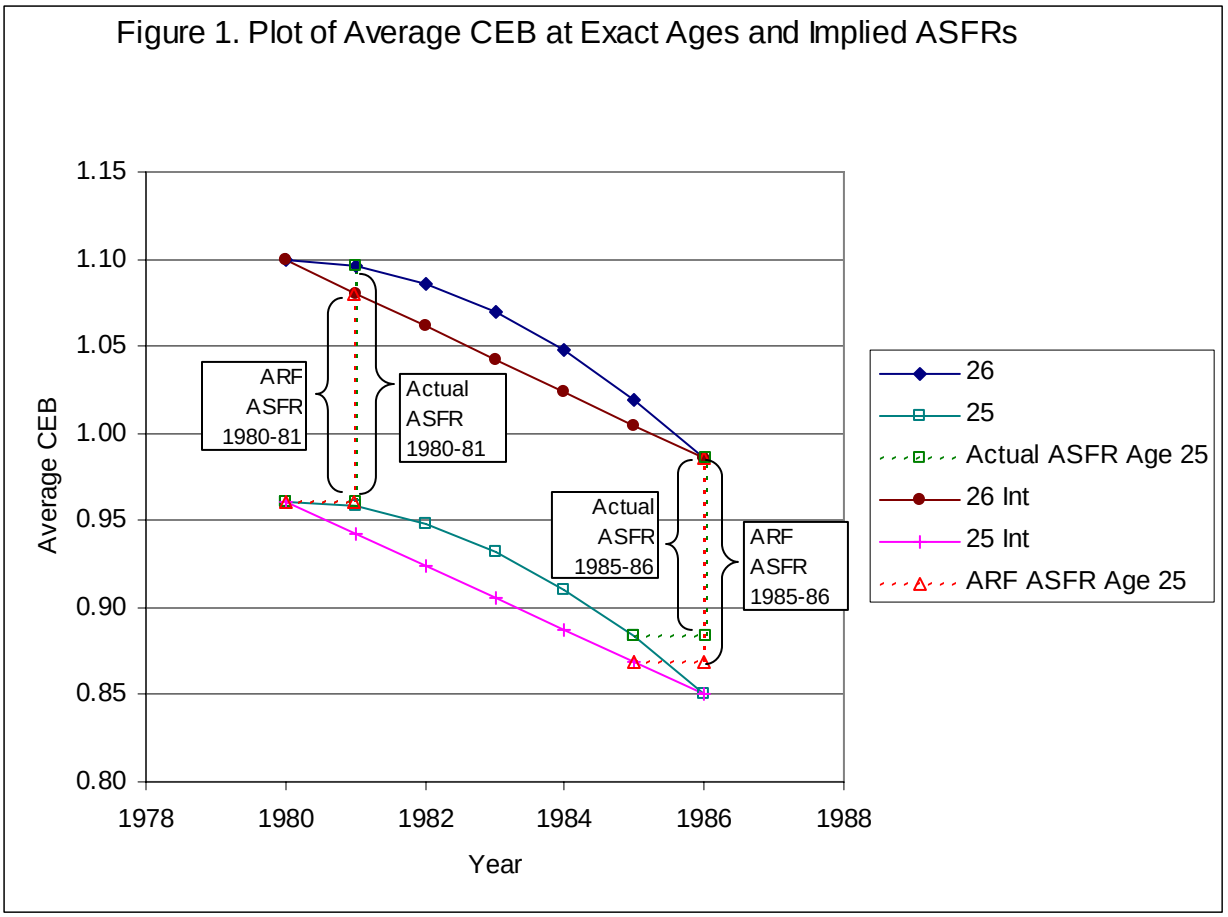
Extended Abstract

Introduction

The Arriaga fertility method (1983) estimates fertility levels by comparing two or more sets of average children ever born (CEB). These estimates are then used to adjust observed fertility patterns in a manner similar to the Brass P/F ratio method. The method was designed to be useful in cases where the Brass P/F ratio method is not appropriate because the level of fertility has been changing. The method was implemented as part of the Population Analysis System (PAS) spreadsheets as ARFE-2 (for two dates) or ARFE-3 (for three dates). This analysis will review the Arriaga method from the theoretical perspective as well as based on simulations. The paper will address the performance of the method, as implemented, under different assumptions regarding fertility change, the effect of fertility under age 15, as well as the impact of the number of decimal places and sample variation in the average parity data.

Assumption of linear CEB change

One of the basic assumptions of the Arriaga fertility method is that the average parity, by age, changes linearly over time. The first part of the paper will examine the validity of this assumption under perfect conditions, when the average parity values by single ages are known without error. A preliminary simple simulation of a rapid linear decline in age-specific fertility rates (ASFRs) indicates that the method tends to measure the average ASFRs over the period between the two sets of CEB data (see figure 1).



In this example, the Arriaga method underestimates the ASFR at the earlier date and overestimates the ASFR at the later date. The curvature of the CEB curves is the result of the cumulation of the ASFRs at earlier years. Based on the mean value theorem, the slope of any continuous function connecting two points will at some point be equal to the slope of the straight line connecting them. It thus appears, at least in this case, that the method might work best near the midpoint between the dates of the two sets of CEB data, rather than near the endpoints.

Impact of under-15 fertility on results of the method

The current version of the Arriaga method creates the equivalent of the Brass P/F ratios, but based on estimates of cumulated ASFRs rather than average parity. This means that the Arriaga comparisons are more consistent, since neither component of the ratio implicitly or explicitly includes under-15 fertility. However, the Arriaga method, as currently implemented, assumes that the CEB curve has a value of zero at age 15.

This means that if the polynomial used to estimate CEB by single ages is fitted to data that includes fertility under age 15, the curve will need to rise faster from age 15 to age 20 in order to get the level of observed CEB for ages 15-19. This will probably result in the value of the polynomial at age 20 being too high, so, in order to reproduce the 20-24 CEB, the curve will have to be lower at age 25. The result could be a highly distorted representation of the CEB curve, and therefore, the estimated ASFRs may be in doubt. Distortions in the single year fitted CEB data have sometimes been observed in practice. This paper will investigate the potential impact of this problem on the estimates, and look for possible changes to correct this problem.

Impact of the number of decimal places of CEB data

Since the method is estimating ASFRs based on differences in CEB data, it seems logical that the number of decimal places of the CEB data may affect the results. Similarly, when the CEB data come from sample surveys, the sample error in the CEB data could imply unacceptably large variation in the resulting ASFR estimates.

Reference dates of input and output ASFRs

The ARFE-2 and ARFE-3 spreadsheets need ASFR (patterns) for the dates near the CEB data. It must be made clear that the method is not adjusting the reported ASFR data in the way the Brass method does. Since the assumption is that fertility may be changing, the pattern of fertility may also be changing. Thus it would seem optimal to include the reference dates for the input ASFRs and interpolate the patterns to the dates needed to compare to the CEB-generated estimates.

References:

U.S. Census Bureau. 1983. “Estimating Fertility From Data on Children Ever Born, by Age of Mother.” By Eduardo Arriaga. International Research Document no. 11. Washington, DC.