Allocating Time:
Individuals' Technologies and Household Technology

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Abstract

The household production model remains the lens through which virtually all economists and many other social scientists view household time allocation. Specialization, especially gender specialization, is a central empirical and theoretical issue. Becker's specialization claim -- that in efficient married couple households, husbands specialize in the market sector and wives in the household sector -- dominates the theoretical landscape.

I argue that specialization conclusions rest on auxiliary assumptions to which neither economic theory in general, nor the household production model in particular, have any commitment. These include assumptions about household technology (e.g., constant or increasing returns to scale; spouses' time inputs are perfect substitutes), household preferences (e.g., the absence of "process benefits"), and the number of "commodities" (e.g., one household commodity).

I focus on the relationship between individuals' technologies and household technology. I show that if the household technology is "additive" (i.e., the sum of the individuals' technologies), then this relationship becomes visible at three crucial life cycle transitions: household formation, divorce, and the death of a spouse.

The household production model remains the lens through which virtually all economists and many other social scientists view household time allocation. For many social scientists, certainly for economists, the starting point of the modern time-use literature is Becker (1965), "A Theory of the Allocation of Time." In that paper, Becker introduced the household production model which has become the centerpiece of what Nerlove (1974) called the "new home economics."[1] It was Becker's 1965 article that placed household time allocation on the agenda for economists and, directly and indirectly, influenced many other social scientists.

Becker (1965) provides a clear statement of the foundations of the new home economics in the context of a single-person household. Becker writes: households are "assumed to combine time and market goods to produce more basic commodities that directly enter their utility functions."[2] Without additional assumptions, the implications of the new home economics for the demand for market goods and for labor supply are essentially equivalent to those of the traditional neoclassical model of consumer behavior.[3] As Pollak and Wachter (1975) argue, this shows that the power of the new home economics to place restrictions on household behavior beyond those implied by neoclassical demand theory depends on imposing further assumptions on household technology or on making assumptions about the relationship between the number of

[1] Of course there were precursors -- most immediately Mincer (1963), and three decades earlier, Reid (1934), *Economics of Household Production*, a book whose title suggests its relationship to Becker's concerns.

[2] Becker (1965) focuses on single-person households, as the phrase "their utility functions" suggests. Only one paragraph discusses "The Division of Labour Within Families."

[3] Proof: Suppose that there are n goods and n commodities, and that the household production functions are such that one unit of good i produces one unit of commodity i; this corresponds to a degenerate household technology in which commodities are produced by market goods without any input of household time. The utility function defined over commodities thus becomes a utility function defined over market goods.

goods and the number of commodities.

For the past 20 years, virtually all research on household time use has been empirical rather than theoretical. The most recent theoretical citation in most empirical papers on time use is Becker (1991), the enlarged edition of his *Treatise on the Family*, and two of the three chapters on household production and time use that appeared in the first edition of the *Treatise* published in 1981. The first of these, a brief chapter on "Single-Person Households," generalized the household production model of Becker (1965) by introducing human capital that augments the productivity of time in household production.[4] The second of these chapters, the "Division of Labor in Households and Families," presents the celebrated specialization theorems. The enlarged edition of 1991 includes an additional chapter on time use that examines the allocation of "effort" and elaborates on the earlier material on the "sexual division of labor."[5] For issues of time allocation within households and families, Becker's *Economic Journal* paper of 1965, Pollak and Wachter's *Journal of Political Economy* paper of 1975, and Becker's *Treatise* 1991 come close to defining the theoretical landscape.[6] [7]

In this paper I explore several themes related to "specialization." I show that Becker's conclusion that husbands specialize in the market and wives specialize in the home does not follow from the basic assumptions of the household production model. Instead, the specialization conclusion rests on auxiliary assumptions to which neither economic theory in general, nor the household production model in particular, have any

[4] The role of human capital in household production was not discussed in Becker (1965).
[5] This chapter originally appeared as Becker (1985).
[6] I discuss Pollak and Wachter (1975) below.
[7] A notable exception to this generalization is Folbre (2004), which provides an accessible critique of the new home economics from a feminist perspective, emphasizing the endogeneity of preferences.

commitment. For example, as Lundberg (2005) points out, the home/market specialization conclusion depends crucially on the assumption that the household "sector" produces only a single "commodity." If we replace Becker's assumption that there is only one household commodity by the alternative assumption that there are m household commodities then, for households in which both husbands and wives participate in the market, Becker's reasoning implies that husbands will specialize in the production of m* of these home-produced commodities and the wives will specialize in the production of the remaining m-m* commodities.

Even this m-commodity specialization conclusion rests on auxiliary assumptions about the household technology. For example, Becker assumes that household production functions exhibit constant or increasing returns to scale. But if individuals who increase the time they devote to an activity become tired or bored, and if fatigue or boredom causes them to become less productive, then the household production functions exhibit decreasing returns to scale.[8] [9] If spouses' production functions for a commodity exhibit decreasing returns to scale, efficiency may require both spouses to participate in its production.

Becker identifies the accumulation of commodity-specific human capital as the primary reason for specialization and the division of labor within the household. I argue that specialization and the division of labor are more likely to reflect other factors, including economies of scope, transaction costs, and bargaining power.

[8] Even if productivity is undiminished, individuals may become less willing to devote additional time and effort to an activity if they become tired or bored. The disutility effects of fatigue and boredom require recognizing "process preferences" -- that is, time allocated to an activity is an argument of the utility function.

[9] Unless this effect is offset by nonlabor inputs becoming more productive as their use increases.

The theory of household production and time use for multiple-person households is seriously underdeveloped.[10] Becker (1965) devotes only a paragraph to multiple-person households; and Pollak and Wachter (1975) completely ignore them. The analysis of time use in multiple-person households involves three basic elements: household technology, individuals' preferences, and household collective choice.

- Household technology specifies the constraints, other than the market and time constraints, that define the household's feasible set. The household technology can be represented by production sets or, in the absence of joint production, by household production functions. In this paper I focus on technology, emphasizing the relationship between household technology and individuals' technologies.
- Individual preferences specify the objective functions that individuals seek to maximize. In multiple-person households, interdependent preferences (e.g., preferences in which each spouse cares about the consumption and time use of the other) seem plausible. In the introduction to the 1991 edition of the *Treatise*, Becker acknowledges that the specification he called "altruism" is overly restrictive because it excludes plausible patterns of interdependence preferences.
- Household collective choice, with its emphasis on bargaining over the division of benefits and burdens within the household, has now moved to

[10] I have written this paper in terms of married couple households, but issues of specialization and division of labor arise in all multiple-person households: cohabiting couples, gay and lesbian couples, or adult children who coreside with disabled elderly parents. Furthermore, as the title of Becker's chapter "Division of Labor in Households and Families" suggests, the issue of specialization implicates families as well as households. How individuals sort themselves into households is beyond the scope of this paper. Ellickson (2006) provides an insightful transaction cost analysis of household size and composition.

center stage in family economics. With their focus on single-person households, neither Becker (1965) nor Pollak and Wachter (1975) discuss household collective choice. In any case, these papers, were written well before the earliest bargaining models.[11] We can reinterpret Becker (1965) and Pollak and Wachter (1975) as applying to multiple-person households that behave as if they were single person households, but the examples these papers offer make it clear that their authors thought in terms of single-person households. Becker (1991) deals with multiple-person households within the confines of a unitary model, more specifically, the "altruist model." Unitary models imply that household behavior is consistent with maximization of a household utility function subject to household resource and technology constraints. An analysis that begins with a unitary model assumes away the bargaining issues that have become central in the economics of the family.

A fourth element, transaction costs, deserves far more attention than it has thus far received. Transaction costs are the costs of negotiating, monitoring, and enforcing agreements. They impose additional constraints on the opportunities available to households and families. Arguably, transaction costs should be treated not as a distinct category but integrated with the analysis of household collective choice.

In addition to household technology, individuals' preferences, household collective choice, and transaction costs, three dynamic factors that operate through

[11] Samuelson (1956) pointed out that the standard, unitary model of the household fails to address the problem of aggregating individuals' preferences into household preferences. I discuss Samuelson's contribution in Pollak (2006). The first bargaining models of marriage were published in the early 1980s by Manser and Brown (1980) and by McElroy and Horney (1981).

technology and through preferences play prominent roles in the analysis of time allocation: household human capital, household physical capital, and preference formation.

In his *Treatise on the Family*, Becker emphasizes the role of human capital in household production.

Recent work in macroeconomics has emphasized the role of physical capital in household production.

Folbre (2004) emphasizes the role of dynamic factors that operate through preferences.

I begin by arguing that the relationship between the household technology and the technologies of the individuals in the households is interesting for its own sake and also interesting because, under plausible assumptions, it implies strong restrictions on household technology. More specifically, I show that, under plausible assumptions about the relationship between household technologies and individuals' technologies, Becker's assumption that spouses' time inputs are perfect substitutes in production is false. I also argue that the relationship between household technology and individuals' technologies is especially interesting at three crucial transition points: household formation, divorce, and the death of a spouse.

- At the point of household formation, the technologies of individuals before they enter the household are related to the technology of the newly-formed household.
- At the point of divorce, the household's technology prior to divorce is related to the technologies of the newly-divorced individuals. In some models of marital bargaining, individuals' well-being in the event of divorce plays the role of a

threat point that determines bargaining power. In other bargaining models, divorce is an "outside option" that determines the range within which any bargaining outcome acceptable to both spouses must lie.

- When a spouse dies, the household's technology prior to the death and the commodity-specific human capital that the surviving spouse acquired during the marriage will determine his or her technology as a widow or widower. Anecdotes about widowers who cannot cook and widows who have never been involved in financial decision making and find themselves confronted by a host of unfamiliar problems exemplify the difficulties that specialization can imply for surviving spouses.[12]

Preliminaries: Single-Person Households

Before discussing multiple-person households, I need to clarify some under-analyzed issues that arise in single-person households and introduce some terminology and notation. For a single-person household, I denote the household production function for commodity z by

$$z = f(t,x)$$

where t denotes the input of time (or "labor") into its production and x the vector of nonlabor inputs. Nonlabor inputs are market goods.[13] The assumption that a technology

[12] Disability of a spouse raises related issues. The nondisabled spouse must either find market substitutes or take over household production activities previously performed by the now disabled spouse. Disability, however, also raises new issues about caregiving.

[13] Household physical capital can, under very special assumptions, be treated like other nonlabor inputs. The assumptions are either perfect rental markets or perfect capital and second-hand markets. When these assumptions are not satisfied, physical capital substantially completes the analysis because the intertemporal budget constraint is not separable by periods.

can be represented by a production function entails two significant restrictions: it presupposes production efficiency and it rules out "joint production."[14] A technology exhibits joint production when it produces two or more outputs.[15] Pollak and Wachter (1975) show that joint production is present whenever individuals have "process" preferences (i.e., "direct" preferences for spending time engaging in some activities and not engaging in others.) For example, if I would rather spend my time cooking than cleaning, then the time I spend cooking and the time I spend cleaning are arguments of my utility function and, hence, are "commodities." "Home cooked meals" and "a clean house" also enter my utility function and, hence, are also commodities. Thus, the activity "cooking" produces both "home cooked meals" and "time spent cooking" and the activity "cleaning" produces both "a clean house" and "time spent cleaning." To deal formally with joint production requires representing technologies by production sets rather than production functions, and describing the allocation of time among "activities" (e.g., "cooking," "cleaning"), where each of these activities produces two commodities. I rely on production functions whenever possible and ignore joint production except when necessary.

Both theoretical and empirical work often assume that the household technology is such that output is proportional to the time input. This assumption can be used to finesse the problem of measuring the commodities produced and plays an important role in motivating specialization results. Proportionality is sometimes interpreted as a property of the household technology, and sometimes as a consequence of efficient

[14] To avoid imposing efficiency, the production function can be reinterpreted as the *maximum* output that can be obtained from the input vector (t,x). The household production literature, however, has not taken this route.

[15] The standard textbook example of joint production is a sheep ranch producing both wool and mutton.

allocation of nonlabor inputs given the technology. The simplest case is one in which time is the only input. When time is the only input, proportionality is equivalent to the assumption that the household production function exhibits constant returns to scale:

$$f(t) = ct.$$

A more general case in which output is proportional to the time input is the Leontief or fixed coefficient production function:

$$f(t,x) = \min \{t/a_1, x/a_2\}.$$

(To simplify the notation, I have written this production function as if there is only one nonlabor input; in the fixed coefficient case, additional inputs pose only notational complications.) If an individual is an efficient producer and has a fixed coefficient technology, then

$$t/a_1 = x/a_2$$

and, hence

$$z = ct$$

where $c = 1/a_1$.

The even more general case of a constant returns to scale production function,

$$f(\lambda t, \lambda y) = \lambda z,$$

is superficially similar but fundamentally different. If the technology exhibits constant returns to scale, if the quantities of all nonlabor inputs are variable, and if the individual is an efficient producer, then output is proportional to the time input. But the time input is an unsatisfactory measure of output because the factor of proportionality depends on the quantities of the nonlabor inputs.[16]

[16] Or, equivalently, on the prices of the nonlabor inputs.

Fixed inputs pose another problem. If some of the inputs are fixed, then the "law of diminishing returns" implies that output will not increase in proportion to the variable inputs. The assumption that all inputs are variable plays a crucial but often unacknowledged role in the reasoning establishing specialization theorems in multiple-person households. The best interpretation of the assumption that all factors are variable is that we are concerned with household behavior in the long run.

An individual who devotes more time to an activity may become tired or bored and, hence, less productive.[17] In this case, even if output is produced by labor alone, an increase in hours worked will not cause a proportionate increase in output. If individuals who devote more time to an activity become less productive, then the household production function exhibits decreasing returns to scale: that is, increasing all inputs, including the time input, by 10% would increase output by less than 10%.

Even if productivity is undiminished, individuals may become less willing to devote additional time and effort to an activity if they become tired or bored. The disutility effects of fatigue and boredom require us to recognize "process preferences" -- that is, time allocated to an activity is an argument of the utility function. The intuition is clear when the utility function is additively separable:

$$U(z_1,...z_m, t_1,...t_m, t_l) = V(z_1,...,z_m) + \Sigma\, v^k(t_k) + v^l(t_l)$$

where t_l is "leisure."[18] The disutility effects of fatigue or boredom imply an increasing marginal disutility of time devoted to the activity.[19]

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[17] The possibility that individuals become less productive as they devote more time to an activity provides the rationale for concern about, and regulation of, the working hours of medical interns and residents, truck drivers, air-traffic controllers, and pilots.

[18] I do not include time allocated to market work, t_w, in the utility function because including it as well as $\{t_1,...,t_m, t_l\}$ would be redundant. The time constraint $\Sigma\, t_k + t_l + t_w = T$, implies that we could rewrite the utility function to include t_w and exclude any one of the other time variables.

[19] This intuition generalizes to indifference curves and marginal rates of substitution.

Productivity and disutility effects are analytically distinct, although they may operate simultaneously.[20] As Pollak and Wachter (1975) point out, the household production literature, often fails to distinguish clearly between production activities that produce outputs and activities that "produce" utility. The failure to distinguish clearly between technology and preferences is especially troubling in the context of multiple-person households.

Household Production in Multiple-Person Households

I begin with notation for the household production function for a commodity, z, ignoring for now process preferences and the pervasiveness of joint production. I denote the household production function for z by $g[t_1,t_2,x]$, where t_1 and t_2 denote the spouses' time inputs into the production of z. I denote the individual production functions of the spouses by $f^1(t_1,y_1)$ and $f^2(t_2,y_2)$. I interpret the individual production functions as those that each spouse would have if the marriage were to end immediately. This interpretation assumes an ongoing marriage; the individual production functions are those that would become operative for the spouses in the event of divorce or for the surviving spouse if one spouse were to die. An alternative interpretation is that $f^1(t_1,y_1)$ and $f^2(t_2,y_2)$ are the production functions of prospective spouses and $g[t_1,t_2,x]$ is the production function of the newly formed household if they were to marry each other.

The relationship between household technology and individuals' technologies is my concern. I say that the household technology for a particular commodity is "additive" if

[20] It is tempting but inaccurate to say that fatigue operates through productivity and boredom through disutility.

$$g[t_1, t_2, x] = \max \{f^1(t_1, y_1) + f^2(t_2, y_2)\}$$

subject to $y_1 + y_2 \leq x$.[21] That is, the household production function is additive if the output the couple would realize from the input vector (t_1, t_2, x) is equal to the maximum of the sum of the outputs they would realize by producing "side by side." Additivity requires the allocation of nonlabor inputs to maximize the sum of outputs and rules out positive and negative externalities associated with side-by-side production.[22] Externalities might make the couple's total output greater or less than the sum of the outputs they could achieve separately. Positive externalities are most plausible in a dynamic setting in which spouses learn from one another. Negative externalities are reflected in the adage, "Too many cooks spoil the broth."[23]

Before discussing further the relationship between individuals' technologies and household technology, I impose a relatively innocuous technical assumption on the individuals' production functions. I assume that when the time input of an individual is 0, then output from that individual's production function is 0:

$$f^1(0,x) = 0 \quad \text{and} \quad f^2(0,x) = 0.$$

In the additive case, this implies

$$g[t_1,0,x] = f^1(t_1,x) \text{ and}$$

[21] This assumes that all of the inputs are "private goods" (i.e., inputs that must be allocated between the spouses); the alternative assumption is that the inputs are household public goods in which case $y_1 = y_2 = x$.

[22] Externalities here refers to "household externalities," that is, externalities within the household.

[23] To formalize within household externalities, I say that the household technology for a particular commodity is "separable" if

$$g[t_1, t_2, x] = \max \varphi[f^1(t_1,y_1), f^2(t_2,y_2)]$$

subject to $y_1 + y_2 \leq x$. With a separable technology, side-by-side production means that $t_1 > 0$ and $t_2 > 0$, where nonlabor inputs are allocated to maximize total output. Separability is a generalization of additivity that is compatible with a restricted type of externalities.

$g[0,t_2,x] = f^2(t_2,x)$.[24]

With an additive technology, if fatigue or boredom affects productivity, then side-by-side production rather than specialization might be efficient.[25] Specialization in the production of a commodity means that only one spouse allocates time to its production. Formally, a household exhibits specialization in the production of the focal commodity if either $t_1 = 0$ or $t_2 = 0$. With an additive technology, the alternative to specialization is side-by-side production. This is most transparent when output is produced by time inputs alone:

$g(t_1,t_2) = f^1(t_1) + f^2(t_2)$.

A two-commodity example in which efficiency requires *nonspecialization* can be constructed by relying heavily on symmetry. Suppose the household produces two commodities with additive technologies. Suppose further that the individual production functions are identical and exhibit decreasing returns to scale. Suppose also that both spouses have identical fixed coefficient preferences, an assumption that fixes the ratio in which a Pareto-efficient household produces the two commodities. Finally, suppose that both spouses allocate at least some time to household production. Under these conditions, it is easy to see that efficiency requires side-by-side production of both commodities.

How does Becker avoid this nonspecialization result? The key assumption is that

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[24] I assume -- and this is an additional assumption -- that a similar relationship holds in the separable case. That is, when the time input of one spouse is 0, then the household production function is equal to the individual production function of the other spouse:

$g[t_1,0,x] = f^1(t_1,y_1)$

and

$g[0,t_2,x] = f^2(t_2,y_2)$.

[25] Side-by-side production rather than specialization might also be Pareto efficient because of the disutility effects of fatigue and boredom.

the time inputs of the spouses are perfect substitutes and constant or increasing returns to scale, although other assumptions play secondary roles. Becker's chapter on multiple-person households focuses on specialization and the division of labor; he presents his results in the form of 5 theorems. To give their flavor, I quote three of these in full:

"Theorem 2.1 If all members of an efficient household have different comparative advantages, no more than one member would allocate time to both the market and household sectors. Everyone with a greater comparative advantage in the market than this member's would specialize completely in the market, and everyone with a greater comparative advantage in the household would specialize completely there."

"Theorem 2.3. At most one member of an efficient household would invest in both market and household capital and would allocate time to both sectors."

"Theorem 2.4 If commodity production functions have constant or increasing returns of scale, *all* members of efficient households would specialize completely in the market or household sectors and would invest only in market or household capital." (italics in original)

Seven points should be noted.

- The theorems are not restricted to married couple or two-adult households, but purport to apply to all multiple-person households.
- The statements of the theorems do not include all of the assumptions. Becker explicitly states some additional assumptions in the nearby text, but other assumptions are left unstated. In Pollak (2003) I argue that this style of presentation -- results presented as "theorems" without explicit statements of their hypotheses -- is vintage Becker: the "Rotten Kid Theorem" is a prime example.

- Human capital -- market human capital and household human capital -- plays a central role. Becker says virtually nothing about household physical capital, but recent work -- especially Greenwood, Seshadri and Yorukoglu (2005) -- has emphasized its importance.
- Efficiency in household production is assumed, sometimes explicitly in the statement of the theorems, sometimes in the surrounding text.
- Some of the theorems assume that there are only two "sectors" -- home and market -- and that these sectors correspond to "commodities." This assumption is crucial for Becker's conclusion about the efficiency of wives specializing in the home and husbands specializing in the market.[26] The assumption that there is only one household commodity is crucial. First, it rules out the possibility that individuals have process preferences, because process preferences would require at least two household commodities. Second, in a world with two household commodities, it might be efficient for the wife to specialize in the production of one commodity and the husband to specialize in the production of the other.
- The assumption that returns to scale are constant or increasing rules out the possibility that an individual who devotes more time to an activity becomes less productive (e.g., as a result of fatigue or boredom). If both spouses experience reduced productivity due to fatigue or boredom and the household technology is additive, then efficiency may require side-by-side production instead of specialization.

[26] At this point in the chapter, Becker's analysis of specialization is gender neutral: he has not yet argued that it is wives who specialize in the home and husbands in the market.

- Sometimes in the text, although not in the statements of the theorems, Becker assumes that the time inputs of husbands and wives are "perfect substitutes" (*Treatise*, p. 32). Neither Becker nor the subsequent literature argues the plausibility of the perfect substitutes assumption. I argue that its role is crucial. Apart from the perfect substitutes assumption and (sometimes) assumptions about returns to scale, Becker says virtually nothing about the technology of married couple households.[27]

The efficiency of households and families is a major theoretical and empirical issue. In the light of Becker's Theorem 2.3 ("At most one member of an efficient household would invest in both market and household capital and would allocate time to both sectors.") a debater might try to score points by the following argument: "We see many households in which husbands and wives participate in both the market sector and the household sector; hence, these households are inefficient." This argument is flawed, but the reasoning is instructive. Becker's specialization conclusion depends on his assumption that there is only one household commodity. With many household commodities, the analogue of Theorem 2.3 does not imply the pattern of specialization between home and market that Becker predicts: in a world with m household commodities, Becker's reasoning implies that one spouse would specialize in m*

[27] Some of the theorems begin with assumptions about comparative advantage rather than assumptions about the underlying household technology.

household activities and the other spouse in the remaining m - m* activities.[28] [29] The empirical literature on household production and time allocation is now looking beyond time allocation between home and market activities to time allocation among specific household activities or tasks; see, for example, Stratton (2005) and Bonke, Deding, Lausten, and Stratton (2006).

No Nonlabor Inputs

The household production and time allocation literature has followed Becker in assuming that the time inputs of husbands and wives are perfect substitutes in production. With no nonlabor inputs, perfect substitutes implies that the household production function is of the form

$$g(t_1,t_2) = g[t_1 + \alpha t_2]$$

where α converts the time input of spouse 2 into units comparable to the time input of spouse 1. Thus, $(t_1 + \alpha t_2)$ is the total time input into the production of the focal commodity, measured in "efficiency units" (i.e., $1/\alpha$ hours of the time of spouse 2 is equivalent to one hour of the time of spouse 1).

[28] Empirical evidence of efficiency or inefficiency within families is very scarce. Udry (1996) found inefficiency in the allocation of family labor between men's and women's farm plots in Burkina Faso, but Akresh (2006) casts doubt on the generalizability of Udry's findings even to other regions of Burkina Faso. Chiappori and his collaborators find no evidence of inefficiency in household expenditure patterns, but the statistical power of these tests is weak. (See, for example, Bourguignon, Browning, Chiappori, and Lechene (1993) and Browning, Bourguignon, Chiappori, and Lechene (1994).) Furthermore, household expenditure patterns are an unlikely place to find evidence of inefficiency. Lundberg and Pollak (2003) argue that inefficiency is most likely to arise in situations in which couples must make big up-front decisions that affect future bargaining power and are unwilling or unable to make binding commitments (e.g., the two- earner couple location problem). Weiss and Willis (1985, 1993) argue that asymmetric information is a likely source of inefficiency in the context of child support by absent fathers.

[29] This assumes that m is the number of household activities operated at strictly positive levels; it does not include activities to which neither spouse allocates time.

When time is the only input, combining Becker's assumption that the spouses' time inputs are perfect substitutes with the assumption that the household technology is additive implies:

$$g(t_1,t_2) = g[t_1 + \alpha t_2] = f^1(t_1) + f^2(t_2).$$

Making use of the assumption that a time input of 0 implies 0 output, we obtain

$$g[t_1] = f^1(t_1)$$

$$g[\alpha t_2] = f^2(t_2).$$

so

$$g[t_1 + \alpha t_2] = g[t_1] + g[\alpha t_2].$$

This is Cauchy's functional equation (see Aczél and Dhombres, 1989). Differentiating with respect to t_1 we obtain:

$$g'[t_1 + \alpha t_2] = g'(t_1).$$

Because t_2 appears on the left hand side but not on the right hand side, the function $g'(\ )$ must be constant (i.e., independent of t_1 and t_2). Hence, $g(\ )$ must be linear in t.[30] Again making use of the assumption that a time input of 0 implies an output of 0 yields

$$g(t) = ct$$

and, hence

$$g[t_1 + \alpha t_2] = c(t_1 + \alpha t_2).$$

This linear specification of the household's technology is unappealing because it is so highly restrictive. It is, however, a straightforward consequence of three

[30] Although the argument in the text depends on differentiability, the result does not.

assumptions:

- the household's technology is additive,
- output is produced by time alone, and
- the time inputs of the spouses are perfect substitutes.

The first two assumptions are consistent with fatigue or boredom causing productivity to decline as more time is allocated to the production of the commodity. For example, the spouses' production functions might be of the form:

$$f^1(t_1) = A_1 (t_1)^{\sigma 1} \quad \text{and} \quad f^1(t_1) = A_2 (t_2)^{\sigma 2}$$

where the parameters σ_1 and σ_2 indicate the returns to scale properties of the spouses' technologies. Imposing the additional assumption that the spouses' time inputs are perfect substitutes implies $\sigma_1 = \sigma_2 = 1$. That is, imposing Becker's perfect substitutes assumption on this particular household technology rules out decreasing returns to scale and implies that both spouses' production functions collapse to the constant returns to scale case, $g(t) = c\ t$.

Returns to scale properties can create incentives for specialization or nonspecialization. Consider three cases.

- Decreasing returns to scale for both spouses implies that doubling the time input of one spouse will cause the output produced by that spouse to less than double. With decreasing returns to scale, production efficiency may require side-by-side production rather than specialization.
- Constant returns to scale is consistent with Becker's assumption that the spouses' time inputs are perfect substitutes.

- Increasing returns to scale implies that doubling the time input of one spouse will cause the output produced by that spouse to more than double. Increasing returns to scale implies productivity incentives for specialization. Of course, the pattern of specialization or nonspecialization depends on the production functions for all commodities and on preferences; it cannot be inferred from the technology for producing a single commodity.

Nonlabor inputs complicate the story. I first consider the case in which all nonlabor inputs are household public goods and then the case in which all nonlabor inputs are private goods that must be allocated between the spouses.[31]

Nonlabor Inputs as Household Public Goods

I begin with the case in which the nonlabor inputs are all household public goods, so both spouses can use them simultaneously. For example, consider a home in which the heating system does not have separate temperature controls in each room, but instead has a single control that imposes the same temperature on all rooms; in this case, the common temperature is a household public good. Denoting nonlabor inputs by x and using the obvious notation, we write $f^1(t_1,x)$ and $f^2(t_2,x)$ and $g[t_1,t_2,x]$. Again starting with the assumption that the household technology is additive, we write

$$g(t_1,t_2,x) = f^1(t_1,x) + f^2(t_2,x).$$

Following Becker, I assume that the time inputs of the spouses are perfect substitutes in

[31] I ignore the case in which some nonlabor inputs are household public goods and others are private goods.

production, so:

$$g(t_1,t_2,x) = g[t_1 + \alpha(x)t_2,x],$$

where I allow α to depend on the vector of nonlabor inputs. (This allows the factor that converts the time input of spouse 2 into units comparable to the time input of spouse 1 to depend on the vector of nonlabor inputs.) Making use of the assumption that a time input of 0 implies 0 output, we obtain

$$g(t_1,x) = f^1(t_1,x)$$

$$g[\alpha(x)t_2,x] = f^2(t_2,x).$$

Hence,

$$g[t_1 + \alpha(x)t_2,x] = g(t_1,x) + g[\alpha(x)t_2,x].$$

Differentiating with respect to t_1 we obtain:

$$g'[t_1 + \alpha(x)t_2,x] = g'(t_1,x).$$

Because t_2 appears on the left hand side but not on the right hand side, the function $g'(t,x)$ must be independent of t_1 and t_2. Hence, $g(t,x)$ must be linear in t.[32] Again making use of the assumption that a time input of 0 implies an output of 0 yields

$$g[t,x] = B(x) + C(x)t.$$

Again making use of the assumption that a time input of 0 implies an output of 0 yields

$$g(t,x) = C(x)t$$

and, hence

$$g[t_1 + \alpha(x)t_2,x] = C(x)\,[t_1 + \alpha(x)t_2].$$

Thus, when nonlabor inputs are household public goods, the implications for the

[32] Although the argument in the text depends on differentiability, the result does not.

household technology and individuals' technologies are similar to the implications in the absence of nonlabor inputs. The crucial difference between these cases is that the factor that converts the time input of spouse 2 into units comparable to the time input of spouse 1 may depend on the nonlabor inputs. Thus, when nonlabor inputs are household public goods, the production functions of the spouses are proportional to their time inputs, where the factor of proportionality may depend on the vector of household public goods.

Nonlabor Inputs as Household Private Goods

I have some preliminary results on the case in which the nonlabor inputs are private goods that must be allocated between the spouses. Let y_1 and y_2 denote the allocation of nonlabor inputs to each spouse, where $y_1 \geq 0$, $y_2 \geq 0$, and $y_1 + y_2 = x$. I assume that nonlabor inputs are allocated efficiently between the spouses, although the efficiency assumption is not innocuous.[33] Assuming that the household production function is additive, we write:

$$g(t_1,t_2,x) = f^1(t_1,y_1) + f^2(t_2,y_2),$$

where y_1 and y_2 denote the efficient allocation of x between the spouses.

Combining the assumption that the household production function is additive with the assumption that the time inputs of the spouses are perfect substitutes, we obtain:

$$g(t_1,t_2,x) = g[t_1 + \alpha t_2,x] = f^1(t_1,y_1) + f^2(t_2,y_2).$$

Again assuming that these technologies have the property that if the labor input is 0, then the output is 0, we obtain

$$g(t,x) = f^1(t,x) \text{ and}$$

[33] The assumption that the constraint $y_1 + y_2 \leq x$ holds as an equality is not entirely innocuous either.

$$g(\alpha t,x) = f^2(t,x).$$

The easy case is one in which α is a constant independent of x. I have not yet characterized the general case in which α depends on x.

That is, provided we measure the time inputs of the spouses in efficiency units, their production functions are identical. For example, if the wife's technology can be represented by a constant returns to scale Cobb-Douglas production function

$$f^1(t,x) = At^{\beta}x^{1-\beta}$$

then the husband's technology must be a constant returns to scale Cobb-Douglas production function with the same coefficient

$$f^2(t,x) = A(\alpha t)^{\beta}y^{1-\beta}.$$

This is an extremely strong restriction on the spouses' individual technologies: the only admissible difference is that an hour of one spouse's time may be equivalent to $1/\alpha$ hours of the time of the other spouse. If the production functions exhibit constant returns to scale, then I have been able to show that efficiency requires the nonlabor inputs to be allocated in proportion to the labor inputs, where the latter are measured in efficiency units. That is,

$$y_1 = t_1x/ (t_1 + \alpha t_2) \text{ and}$$

$$y_2 = \alpha t_2x/ (t_1 + \alpha t_2).$$

I suspect that constant returns to scale or some closely-related restriction is implied by the assumptions that the household production function is additive and that the time inputs of the spouses are perfect substitutes, but I have not yet been able to prove it.

Conclusion

The household production model remains the lens through which many social scientists view time allocation within households. Specialization, especially gender specialization, is a central empirical and theoretical issue. Becker's specialization claim -- in efficient married couple households, husbands specialize in the market sector and wives in the household sector -- dominates the theoretical landscape.

I argue that specialization conclusions rest on auxiliary assumptions to which neither economic theory in general, nor the household production model in particular, have any commitment. These include assumptions about household preferences (e.g., the absence of "process benefits"), household technology (e.g., spouses' time inputs are perfect substitutes), and the number of "commodities" (e.g., one household commodity).

I say that a household technology is "additive" if it is the sum of the individuals' technologies. Additivity is a simple and plausible assumption about the relationship between individuals' technologies and household technology. Decreasing returns to scale is also a plausible assumption about individuals' technologies: if individuals who increase the time they devote to an activity become tired or bored, and if this causes them to become less productive, then their production functions exhibit decreasing returns to scale. If household technology is additive and exhibits decreasing returns to scale, I show that production efficiency may require spouses to engage in side-by-side production rather than to specialize.

Additivity and Becker's assumption that spouses' time inputs are perfect substitutes are compatible only in a very narrow class of cases. More specifically, if time is the only input into household production, then additivity and perfect substitutes imply

that individuals' production functions and household production functions are linear in the spouses' time inputs.

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